

FIGURE 6-18

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Effect of Mean Stress on Fatigue Life (Data from Fig. 5.10, p. 73, Fuchs and Stephens, *Metal Fatigue in Engineering*, New York, 1980)

Fracture-Mechanics Criteria

The static fracture toughness test was described in Section 5.3. To develop fatigue strength data in terms of fracture-mechanics theory, a number of specimens of the same material are tested to failure at various levels of cyclical stress range $\Delta\sigma$. The test is done in an axial fatigue machine and the load pattern is usually either repeated or fluctuating tensile stresses as shown in Figure 6-6b and 6-6c. Reversed-stress tests are seldom done for these data, since compressive stress does not promote crack growth. The crack growth is continuously measured during the test. The applied stresses range from σ_{min} to σ_{max} . A **stress intensity factor range** ΔK can be calculated for each fluctuating-stress condition from

$$\Delta K = K_{max} - K_{min} : \quad \text{if } K_{min} < 0 \quad \text{then } \Delta K = K_{max} \quad (6.3a)$$

Substituting the appropriate equation 5.14 gives:

$$\begin{aligned} \Delta K &= \beta\sigma_{max}\sqrt{\pi a} - \beta\sigma_{min}\sqrt{\pi a} \\ &= \beta\sqrt{\pi a}(\sigma_{max} - \sigma_{min}) \end{aligned} \quad (6.3b)$$

The log of the rate of crack growth as a function of cycles da/dN is calculated and plotted versus the log of the stress intensity factor range ΔK as shown in Figure 6-19.

The sigmoidal curve of Figure 6-19 is divided into three regions labeled I, II, and III. Region I corresponds to the crack initiation stage, region II to the crack growth (crack propagation) stage, and region III to unstable fracture. Region II is of interest in predicting fatigue life, and that part of the curve is a straight line on log coordinates. Paris^[13] defined the relationship in region II as

$$\frac{da}{dN} = A(\Delta K)^n \quad (6.4a)$$

Barson^[14] tested a number of steels and developed empirical values for the coefficient A and the exponent n in equation 6.4. These are shown in Table 6-2. The fatigue crack

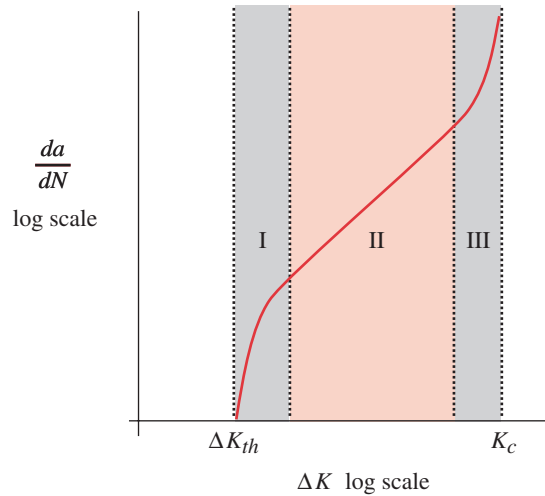


FIGURE 6-19

Three Regions of the Crack Growth-Rate Curve (Adapted from Fig. 3-12, p. 102, in Bannantine et al., *Fundamentals of Metal Fatigue Analysis*, Prentice-Hall, Englewood Cliffs, N.J., 1990, with permission)

growth life is found by integrating equation 6.4a between a known or assumed initial crack length and a maximum acceptable final crack length based on the particular load, geometry, and material parameters for the application. The number of cycles N to grow a crack from an initial size a_i to a given size a_f under a known stress range cycle $\Delta\sigma$ and geometry β can be estimated from the Paris equation parameters as shown in Equation 6.4b:*

$$N = \frac{a_f^{(1-n/2)} - a_i^{(1-n/2)}}{A\beta^n \pi^{n/2} \Delta\sigma^n (1-n/2)} \quad (6.4b)$$

* See Section 5.3 for a definition of the geometry factor β .

Region I in Figure 6-19 is also of interest, since it shows the existence of a minimum threshold ΔK_{th} below which no crack growth will occur. This “*threshold stress intensity factor ΔK_{th} has often been considered analogous to the unnotched fatigue limit S_e , since an applied stress intensity factor range ΔK below ΔK_{th} does not cause fatigue crack growth.*”^[15]

These axial fatigue tests have a mean stress component present, and the level of mean stress has an effect on the rate of crack propagation. Figure 6-20 shows a schematic set

Table 6-2 Paris-Equation Parameters for Various Steels (Equation 6.4a)

Steels	SI units			U.S. (ips) units		
	A	n	Units	A	n	Units
Ferritic-Pearlitic	$6.90E-12$	3.00	$\left[\frac{\text{mm/cycles}}{\text{MPa}^n (\sqrt{\text{m}})^n} \right]$	$3.60E-10$	3.00	$\left[\frac{\text{in/cycles}}{\text{kpsi}^n (\sqrt{\text{in}})^n} \right]$
Martensitic	$1.35E-10$	2.25		$6.60E-09$	2.25	
Austenitic Stainless	$5.60E-12$	3.25		$3.00E-10$	3.25	

Data from Reference 14.