$$y = \frac{1}{EI}\left( \begin{array}{l} \frac{R_1}{6}x^3 + \frac{1}{b}\left( \frac{w}{24}\langle b-a\rangle^4 - \frac{R_1}{6}b^3 \right)x - \frac{w}{24}\langle x-a\rangle^4 \\ \quad + \frac{R_2}{6}\langle x-b\rangle^3 + \frac{R_3}{6}\langle x-l\rangle^3 \end{array} \right) \qquad (k)$$

7 Plots of the loading, shear, moment, slope, and deflection functions are shown in Figure 4-27 and their extreme values in Table 4-2. The files EX04-07 can be opened in the program of your choice to examine the model and see larger-scale plots of the functions shown in Figure 4-27.

This example shows that singularity functions provide a good way to solve beam problems for reactions and deflections simultaneously when there are redundant reactions present. The singularity functions allow the writing of a single expression for each function that applies across the entire beam. They also are inherently computerizable in conjunction with an equation solver that will solve simultaneous equations. The singularity function method presented here is universal and will solve any problems of the types presented.

There are other techniques for the solution of deflection and redundant reaction problems. **Finite element analysis** (FEA) will solve these problems (see Chapter 8). The **area-moment method** treats the moment function as if it were a "loading" function and integrates twice to obtain the deflection function. The reader is referred to this chapter's bibliography for additional information on these topics. **Castigliano's method** uses strain energy equations to determine the deflection at any point.

**Table 4-2**
Example 4-7
Calculated Data

| Variable | Value | Unit |
|---|---|---|
| $R_1$ | 158.4 | lb |
| $R_2$ | 2471.9 | lb |
| $R_3$ | 369.6 | lb |
| $C_1$ | 0.0 | lb |
| $C_2$ | 0.0 | lb-in |
| $C_3$ | –1052.7 | lb-in$^2$ |
| $C_4$ | 0.0 | lb-in$^3$ |
| $V_{min}$ | –1291.6 | lb |
| $V_{max}$ | 1130.4 | lb |
| $M_{min}$ | –1141.1 | lb-in |
| $M_{max}$ | 658.7 | lb-in |
| $\theta_{min}$ | –0.025 | deg |
| $\theta_{max}$ | 0.027 | deg |
| $y_{min}$ | –0.0011 | in |
| $y_{min}$ | 0.0001 | in |

## 4.11 CASTIGLIANO'S METHOD

Energy methods often provide simple and rapid solutions to problems. One such method useful for the solution of beam deflections is that of Castigliano. It can also provide a solution to indeterminate beam problems. When an elastic member is deflected by the application of a force, torque, or moment, strain energy is stored in the member. For small deflections of most geometries, the relationship between the applied force, moment, or torque and the resulting deflection can be assumed to be linear as shown in Figure 3-42, repeated here. This relationship is often called the spring rate $k$ of the system. The area under the load deflection curve is the strain energy $U$ stored in the part. For a linear relationship, this is the area of a triangle:

$$U = \frac{F\delta}{2} \qquad (4.20)$$

where $F$ is the applied load and $\delta$ is the deflection.

Castigliano observed that when a body is elastically deflected by any load, the deflection in the direction of that load is equal to the partial derivative of the strain energy with respect to that load. Letting $Q$ represent a generalized load and $\Delta$ represent a generalized deflection,

$$\Delta = \frac{\partial U}{\partial Q} \qquad (4.21)$$