Equations 13.1 (p. 668) can easily be solved with a computer for all values of ωt needed. But, it is rather difficult to look at equation 13.1f and visualize the effects of changes in the design parameters r and l on the acceleration. It would be useful if we could derive a simpler expression, even if approximate, that would allow us to more easily predict the results of design decisions involving these variables. To do so, we will use the binomial theorem to expand the radical in equation 13.1d for piston position to put the equations for position, velocity, and acceleration in simpler, approximate forms which will shed some light on the dynamic behavior of the mechanism.

The general form of the binomial theorem is:

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3!}a^{n-3}b^3 + \cdots \tag{13.2a}$$

The radical in equation 13.1d is:

$$\sqrt{1-\left(\frac{r}{l}\sin\omega t\right)^2} = \left[1-\left(\frac{r}{l}\sin\omega t\right)^2\right]^{\frac{1}{2}} \tag{13.2b}$$

where, for the binomial expansion:

$$a = 1 \qquad b = -\left(\frac{r}{l}\sin\omega t\right)^2 \qquad n = \frac{1}{2} \tag{13.2c}$$

It expands to:

$$1 - \frac{1}{2}\left(\frac{r}{l}\sin\omega t\right)^2 - \frac{1}{8}\left(\frac{r}{l}\sin\omega t\right)^4 - \frac{1}{16}\left(\frac{r}{l}\sin\omega t\right)^6 - \cdots \tag{13.2d}$$

or:

$$1 - \left(\frac{r^2}{2l^2}\right)\sin^2\omega t - \left(\frac{r^4}{8l^4}\right)\sin^4\omega t - \left(\frac{r^6}{16l^6}\right)\sin^6\omega t - \cdots \tag{13.2e}$$

Each nonconstant term contains the **crank-conrod ratio** r/l to some power. Applying some engineering common sense to the depiction of the slider-crank in Figure 13-7a (p. 669), we can see that if r/l were greater than 1 the crank could not make a complete revolution. In fact if r/l even gets close to 1, the piston will hit the fixed pivot O_2 before the crank completes its revolution. If r/l is as large as 1/2, the transmission angle $(\pi/2 - \phi)$ will be too small (see Sections 3.3, p. 100 and 4.11, p. 204) and the linkage will not run well. A practical upper limit on the value of r/l is about 1/3. Most slider-crank linkages will have this **crank-conrod ratio** somewhere between 1/3 and 1/5 for smooth operation. If we substitute this practicl upper limit of $r/l = 1/3$ into equation 13.2e, we get:

$$1 - \left(\frac{1}{18}\right)\sin^2\omega t - \left(\frac{1}{648}\right)\sin^4\omega t - \left(\frac{1}{11\,664}\right)\sin^6\omega t - \cdots$$

or:

$$1 - 0.055\,56\sin^2\omega t - 0.001\,54\sin^4\omega t - 0.000\,09\sin^6\omega t - \cdots \tag{13.2f}$$

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