

**FIGURE 13-6**

Gas force functions in the two-stroke and four-stroke cycle engines

$$\begin{aligned} q &= r \sin \theta = l \sin \phi \\ \theta &= \omega t \end{aligned} \quad (13.1a)$$

$$\begin{aligned} \sin \phi &= \frac{r}{l} \sin \omega t \\ s &= r \cos \omega t \\ u &= l \cos \phi \\ x &= s + u = r \cos \omega t + l \cos \phi \end{aligned} \quad (13.1b)$$

$$\cos \phi = \sqrt{1 - \sin^2 \phi} = \sqrt{1 - \left(\frac{r}{l} \sin \omega t \right)^2} \quad (13.1c)$$

$$x = r \cos \omega t + l \sqrt{1 - \left(\frac{r}{l} \sin \omega t \right)^2} \quad (13.1d)$$

Equation 13.1d is an exact expression for the piston position x as a function of r , l , and ωt . This can be differentiated versus time to obtain exact expressions for the velocity and acceleration of the piston. For a steady-state analysis we will assume ω to be constant.

$$\dot{x} = -r\omega \left[\sin \omega t + \frac{r}{2l} \frac{\sin 2\omega t}{\sqrt{1 - \left(\frac{r}{l} \sin \omega t \right)^2}} \right] \quad (13.1e)$$

$$\ddot{x} = -r\omega^2 \left[\cos \omega t - \frac{r \left[l^2 (1 - 2 \cos^2 \omega t) - r^2 \sin^4 \omega t \right]}{\left[l^2 - (r \sin \omega t)^2 \right]^{\frac{3}{2}}} \right] \quad (13.1f)$$