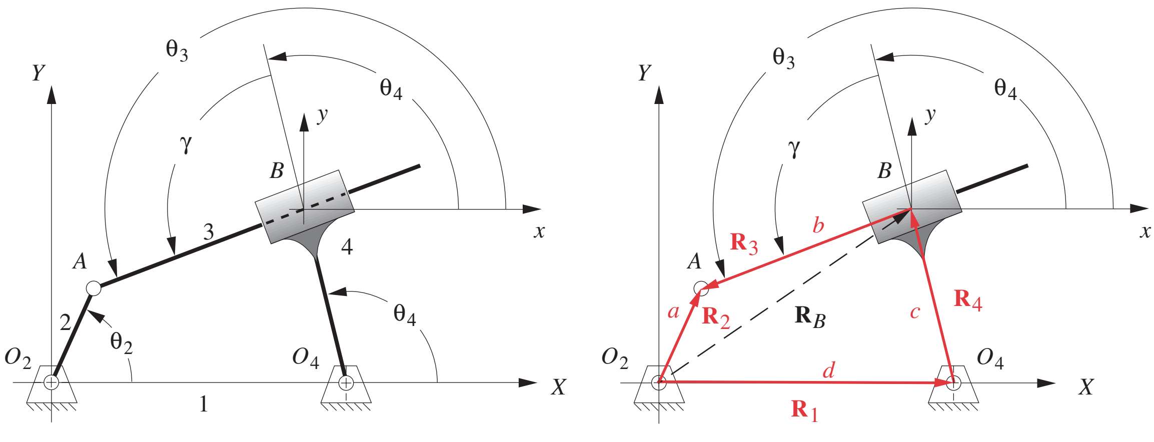

(a) (b)

FIGURE 4-13

Inversion #3 of the slider-crank fourbar linkage

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Equations 4.14 and 4.15 (pp. 192–193) apply to this inversion as well. Note that the absolute position of point B is defined by vector $\mathbf{R}_B$ which varies in both magnitude and direction as the linkage moves. We choose to represent $\mathbf{R}_B$ as the vector difference $\mathbf{R}_2 - \mathbf{R}_3$ in order to use the actual links as the position vectors in the loop equation.

All slider linkages will have at least one link whose effective length between joints will vary as the linkage moves. In this example the length of link 3 between points A and B, designated as b, will change as it passes through the slider block on link 4. Thus the value of b will be one of the variables to be solved for in this inversion. Another variable will be θ_4, the angle of link 4. Note however, that we also have an unknown in θ_3, the angle of link 3. This is a total of three unknowns. Equations 4.15 can only be solved for two unknowns. Thus we require another equation to solve the system. There is a fixed relationship between angles θ_3 and θ_4, shown as γ in Figure 4-13, which gives the equations for the open and crossed configurations of the linkage, respectively:

$$\text{open configuration:}\quad \theta_3 = \theta_4 + \gamma; \qquad \text{crossed configuration:}\quad \theta_3 = \theta_4 + \gamma - \pi \tag{4.22}$$

Repeating equations 4.15 and renumbering them for the reader's convenience:

$$a\cos\theta_2 - b\cos\theta_3 - c\cos\theta_4 - d = 0 \tag{4.23a}$$

$$a\sin\theta_2 - b\sin\theta_3 - c\sin\theta_4 = 0 \tag{4.23b}$$

These have only two unknowns and can be solved simultaneously for θ_4 and b. Equation 4.23b can be solved for link length b and substituted into equation 4.23a.

$$b = \frac{a\sin\theta_2 - c\sin\theta_4}{\sin\theta_3} \tag{4.24a}$$

$$a\cos\theta_2 - \frac{a\sin\theta_2 - c\sin\theta_4}{\sin\theta_3}\cos\theta_3 - c\cos\theta_4 - d = 0 \tag{4.24b}$$

Substitute equation 4.22 and after some algebraic manipulation, equation 4.24 can be reduced to:

$$P\sin\theta_4 + Q\cos\theta_4 + R = 0 \tag{4.25}$$

where

$$P = a\sin\theta_2\sin\gamma + \left(a\cos\theta_2 - d\right)\cos\gamma$$
$$Q = -a\sin\theta_2\cos\gamma + \left(a\cos\theta_2 - d\right)\sin\gamma$$
$$R = -c\sin\gamma$$

Note that the factors *P, Q, R* are constant for any input value of θ_2. To solve this for θ_4, it is convenient to substitute the tangent half angle identities (equation 4.9, p. 188) for the sin θ_4 and cos θ_4 terms. This will result in a quadratic equation in tan $(\theta_4 / 2)$ which can be solved for the two values of θ_4.

$$P\frac{2\tan\left(\frac{\theta_4}{2}\right)}{1+\tan^2\left(\frac{\theta_4}{2}\right)} + Q\frac{1-\tan^2\left(\frac{\theta_4}{2}\right)}{1+\tan^2\left(\frac{\theta_4}{2}\right)} + R = 0 \tag{4.26a}$$

This reduces to:

$$\left(R-Q\right)\tan^2\left(\frac{\theta_4}{2}\right) + 2P\tan\left(\frac{\theta_4}{2}\right) + \left(Q+R\right) = 0$$

let

$$S = R - Q, \qquad T = 2P, \qquad U = Q + R$$

then

$$S\tan^2\left(\frac{\theta_4}{2}\right) + T\tan\left(\frac{\theta_4}{2}\right) + U = 0 \tag{4.26b}$$

and the solution is:

$$\theta_{4_{1,2}} = 2\arctan\left(\frac{-T \pm \sqrt{T^2 - 4SU}}{2S}\right) \tag{4.26c}$$

As was the case with the previous examples, this also has a crossed and an open solution represented by the plus and minus signs on the radical. Note that we must also calculate the values of link length *b* for each θ_4 by using equation 4.24a. he coupler angle θ_3 is found from equations 4.22 for the open or crossed solution.

4.9 LINKAGES OF MORE THAN FOUR BARS

With some exceptions,* the same approach as shown here for the fourbar linkage can be used for any number of links in a closed-loop configuration. More complicated linkages may have multiple loops which will lead to more equations to be solved simultaneously and may require an iterative solution. Alternatively, Wampler [10] presents a new, general, noniterative method for the analysis of planar mechanisms containing any number of rigid links connected by rotational and/or translational joints.

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* Waldron and Sreenivasan[1] report that the common solution methods for position analysis are not general, i.e., are not extendable to *n*-link mechanisms. Conventional position analysis methods, such as those used here, rely on the presence of a fourbar loop in the mechanism that can be solved first, followed by a decomposition of the remaining links into a series of dyads. Not all mechanisms contain fourbar loops. (One eightbar, 1-*DOF* linkage contains no fourbar loops—see the 16th isomer at lower right in Figure 2-11d on p. 50). Even if there is a fourbar loop, its pivots may not be grounded, requiring that the linkage be inverted to start the solution. Also, if the driving joint is not in the fourbar loop, then interpolation is needed to solve for link positions.