uses the elliptical curve of Figure 10-3 fitted through the bending endurance strength on the σ_a axis and the **tensile yield strength** on the σ_m axis as the failure envelope. The tensile yield strength is substituted for the torsional yield strength by using the von Mises relationship of equation 5.9 (p. 251). The derivation of the ASME shaft equation is as follows.

Starting with the relationship for the failure envelope shown in Figure 10-3*a*:

$$\left(\frac{\sigma_a}{S_e}\right)^2 + \left(\frac{\tau_m}{S_{ys}}\right)^2 = 1 \tag{10.5a}$$

introduce a safety factor N_f

$$\left(N_f \frac{\sigma_a}{S_e}\right)^2 + \left(N_f \frac{\tau_m}{S_{ys}}\right)^2 = 1 \tag{10.5b}$$

Recall the von Mises relationship for S_{ys} from equation 5.9 (p. 251):

$$S_{ys} = S_y \big/ \sqrt{3} \tag{10.5c}$$

and substitute it in equation 10.5*b*.

$$\left(N_f \frac{\sigma_a}{S_e}\right)^2 + \left(N_f \sqrt{3} \frac{\tau_m}{S_y}\right)^2 = 1 \tag{10.5d}$$

Substitute the expressions for σ_a and τ_m from equations 10.2*c* and 10.3*c*, respectively:

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$$\left[\left(K_f \frac{32M_a}{\pi d^3}\right)\left(\frac{N_f}{S_e}\right)\right]^2 + \left[\left(K_{fsm} \frac{16T_m}{\pi d^3}\right)\left(\frac{N_f \sqrt{3}}{S_y}\right)\right]^2 = 1 \tag{10.5e}$$

which can be rearranged to solve for the shaft diameter d as

$$d = \left\{ \frac{32N_f}{\pi} \left[\left(K_f \frac{M_a}{S_f}\right)^2 + \frac{3}{4}\left(K_{fsm} \frac{T_m}{S_y}\right)^2 \right]^{\frac{1}{2}} \right\}^{\frac{1}{3}} \tag{10.6a}$$

The notation used in equation 10.6 is slightly different than that of the ANSI/ASME standard in order to remain consistent with the notation used in this text. The standard uses the approach of reducing the fatigue strength S_f by the fatigue-stress-concentration factor K_f rather than using K_f as a stress increaser as is done consistently in this text. In most cases (including this one) the result is the same.* Also, the ASME standard assumes the stress concentration for mean stress K_{fsm} to be 1 in all cases, which gives

$$d = \left\{ \frac{32N_f}{\pi} \left[\left(K_f \frac{M_a}{S_f}\right)^2 + \frac{3}{4}\left(\frac{T_m}{S_y}\right)^2 \right]^{\frac{1}{2}} \right\}^{\frac{1}{3}} \tag{10.6b}$$

* Note that it is better practice to use stress concentration factors to increase the local stresses rather than to decrease material strength because that allows application of the proper (and different) stress concentration factors to the tensile and shear stress components and gives a more accurate result.