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OPTIMIZATION OF PARAMETERS FOR SPECIFIED PATH GENERATION USING AN ATLAS OF COUPLER CURVES OF GEARED FIVE-BAR LINKAGES

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Abstract—An atlas containing 732 coupler curves traced by the symmetric geared five-bar linkage has been drawn by the authors. Gear ratios of +1, −1, +2, −2, have been used. The data are presented either six or twelve curves to a page, arranged in axonometric projection to illustrate the envelope of a family of curves of that particular linkage geometry as the gearset phase angle is varied. Dots on the curve represent uniform intervals of input crank angular displacement in order to provide coupler point velocity information. Other curves for parameters lying between those illustrated can be visually interpolated. Optimization techniques can be applied to modify the geometric parameters of a linkage selected from the atlas as an approximate solution to the path motion desired. The original curves are drawn on 11 × 17 sheets in four colors with a Benson–Varian *xy* plotter. Optimization algorithms are then used in conjunction with the atlas of linkage coupler curves to refine an initial selection of a desired motion path to achieve a particular motion with minimum deviation. The geared five-bar linkage is used as an example and a straight line generator is designed with less than 1% position error.

1. INTRODUCTION

Geared five-bar mechanisms (GFBM) have been known for many years. Reuleaux[1] examined various inversions of the GFBM in 1875 and cited some applications of that era such as sewing machine needle mechanisms, straight line motions, steering mechanisms and stamping machines. Many other authors have discussed and analyzed this device. Tao and Hall[2] analyzed the symmetrical geared five-bar linkage for various gear ratios and phase angles. Freudenstein and Primrose[3, 4] have derived the equations for the coupler curves generated by the joint between the two floating links and defined the general properties of the coupler curve for any geared five-bar linkage. The GFBM can be considered to be a modification of Stephenson's 6-link, one freedom chain in which one binary link (link 6 in Fig. 1a) is replaced by a two-freedom joint in the form of a gear mesh. The gear teeth are part of links 2 and 5, respectively (Fig. 1b). A more common embodiment is shown in Fig. 1(c) in which complete gears containing joints I_{23} and I_{45} allow complete revolution of links 2 and 5, with input provided to either. In most of the GFBM literature, the joint I_{34} is used as the coupler point, although any points on the two coupler planes formed by link 3 and 4 extended can serve as a source of output motion.

Another perspective from which to view the GFBM is to consider it a modification of the five-bar binary chain, which is a two degree of freedom device requiring two simultaneous inputs for constraint (see Fig. 2a). By coupling two of the links together with a gearset, a single input to any link results in a coupled input to a second link through the gearset, thus reducing the degree-of-freedom (DOF) to +1 (Fig. 2b).

If the gearset possesses a ratio of plus 1, requiring an idler, (Fig. 2c), the coupler curves generated become identical to those generated by another four-bar linkage which is not necessarily a Grashoff crank rocker. For a ratio of minus 1, the coupler curves generated will differ from those of a four-bar of similar geometry. For ratios other than one the coupler curve will be of higher order than sixth and will have multiple loops[3]. Tao[5] has shown the application of multi-loop GFBM coupler curves to the design of multiple dwell mechanisms without cams. Rose[6] discusses the design of GFBM solutions to problems of pseudo-straight line generation and precision point curve generation.

While the commonly used four-bar linkage[16] has great versatility in terms of the kinds of path and function generation obtainable, the geared five-bar introduces several additional design parameters, namely: gear ratio, phase angle and additional link ratio. In addition, it is not necessary that the gear ratio be a constant. Non-circular gears, or their drag link 4-bar equivalent, can be used to couple the "geared" links, thus allowing the displacement and velocity profile of the coupler point to be further varied over that obtained with constant ratio. In the case of a drag link coupling, the linkage reverts to a Stephenson's 6-bar (Fig. 1a).

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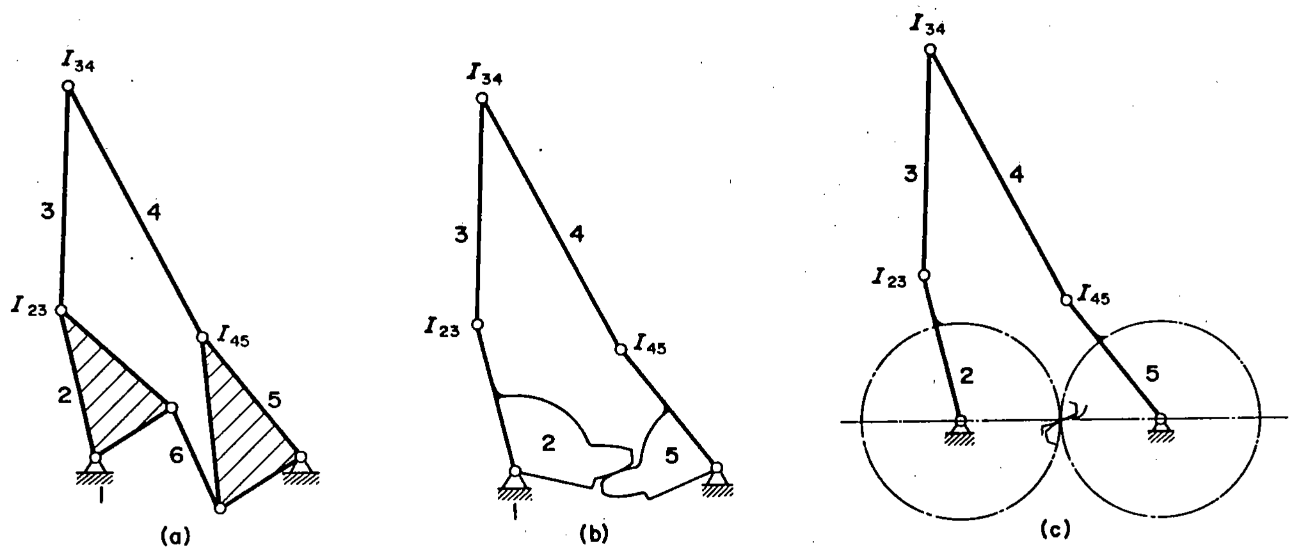


Fig. 1. Relation between GFBM and Stephenson's 6-bar mechanism.

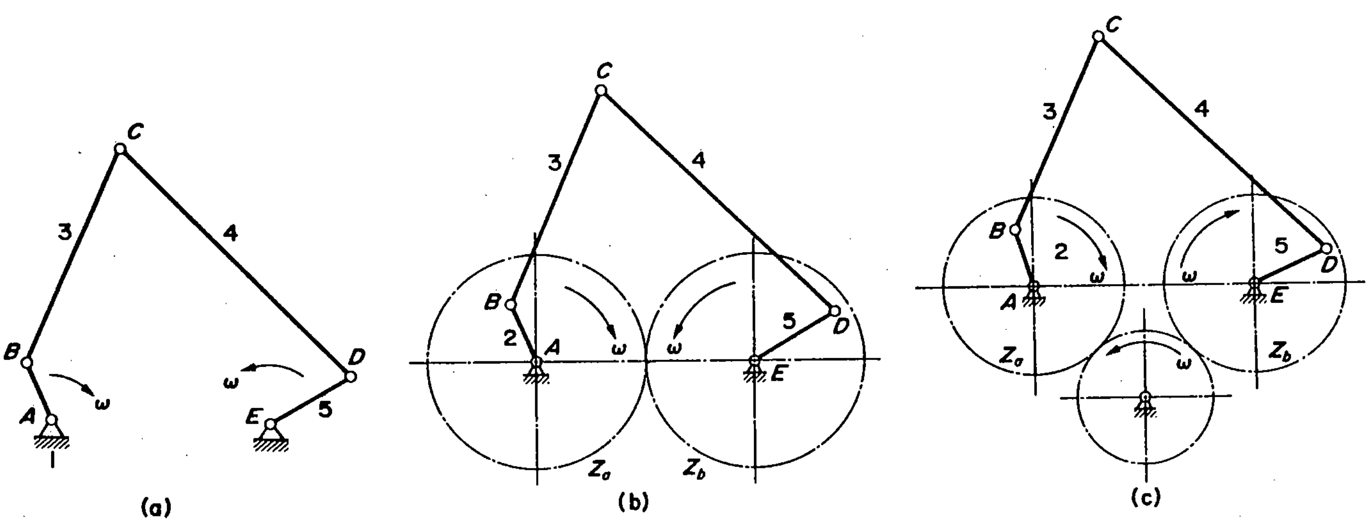


Fig. 2. Non-geared and geared five-bar mechanisms.

2. ARRANGEMENT OF THE ATLAS

For simplicity in plotting and convenience in use, a concise atlas is desired. To limit the number of combinations of parameters, the following measures have been taken: (1) only symmetric linkages are considered; (2) only the pin connecting the two couplers is taken as the tracer-point; (3) only the integers ± 1, ± 2 are used as the gear ratio; (4) consideration is given to symmetry and repeatability of the curves in determining the range of phase angles displayed. Based on these simplifications, the number of basic parameters of a symmetric GFBM path-generator is reduced to 4 (Fig. 3): λ–gear ratio, α–ratio of length of the floating couplers over that of the cranks, β–ratio of length of the center distance connecting the two fixed pivots over that of the cranks, γ-phase angle.

The presentation of the coupler curve data is designed to show the "envelope" of a family of coupler curves when a single design parameter such as phase angle is varied. It is quite likely that a curve lying between those displayed may be more nearly optimal for a given design. Further, the atlas is intended to be used to identify a starting design which appears close to the path and/or velocity profile desired. The design can then be modified using an optimization algorithm such as DFP, or POWELL[10–15] to change any or all of the linkage parameters and improve the design. Although a symmetric linkage is used as the starting design, usually an unsymmetric linkage is obtained after the optimization process, with the tracer-point not necessarily still being the pin between the two couplers[17].

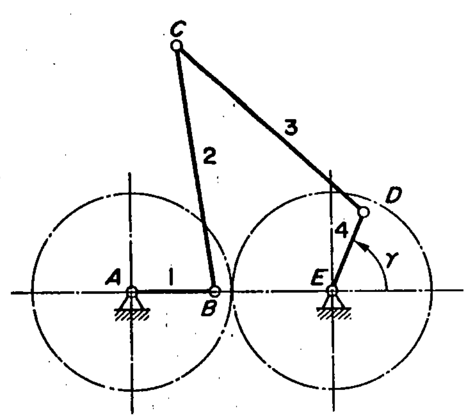
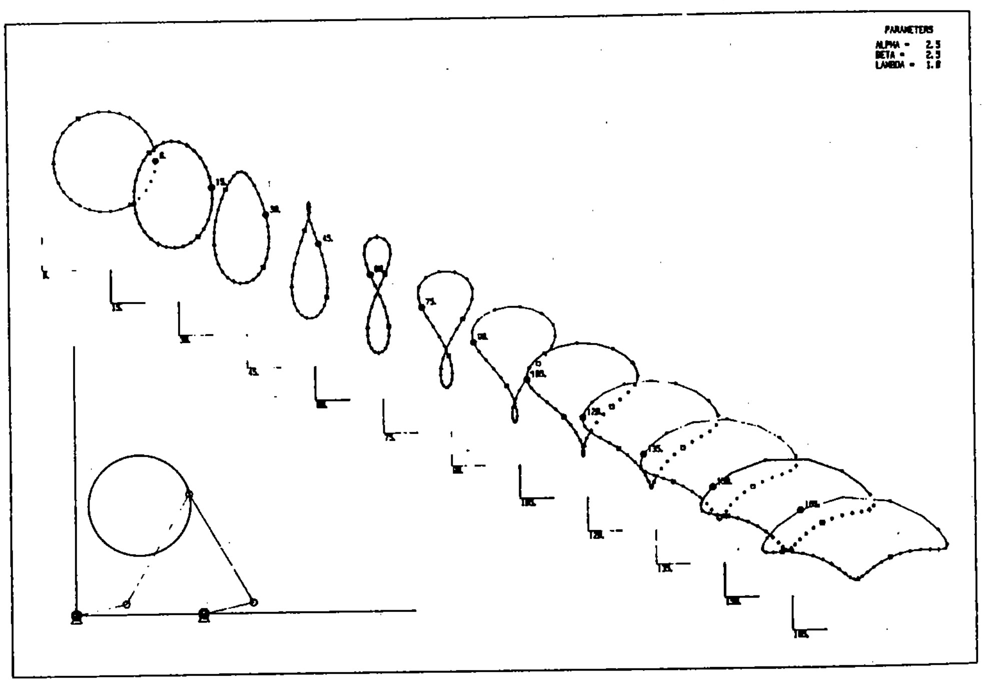


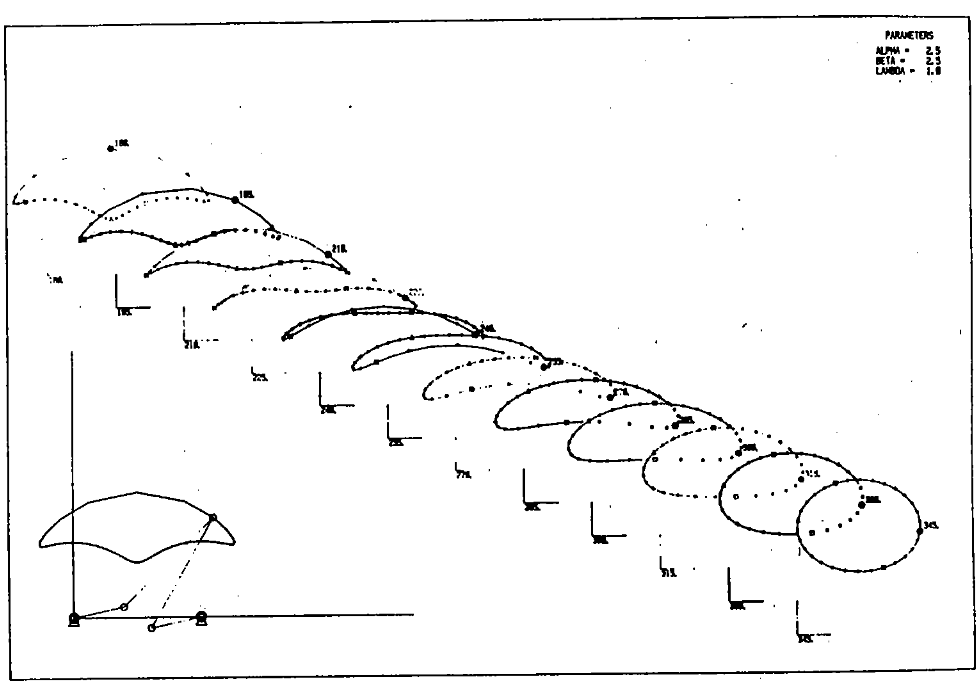
Fig. 3. Symmetric GFBM.

Approximately 732 coupler curves are accurately plotted for a variety of link ratios and phase angles. The maximum position error produced by the Benson–Varian plotter is about 0.005 ins. Each coupler curve has 30 dots, or nodes. The distance between adjacent nodes measured along the curve is the displacement of the tracer-point for a $|12\lambda|$ degree angular displacement of the driving crank. The family of curves generated by a linkage with the same value of α, β, λ but different phase angle γ are shown on the same page of the atlas and put in X–Y planes parallel to each other. The origins of these X–Y coordinate frames are arranged along a third skew axis, which can be called the "phase angle axis". In such a three-dimensional representation, a curved surface, or "envelope", is implied and the trend of variation of the curves can be observed. One can infer any curve between two given curves by interpolation and use it as a better starting design.

Figures 4–7 are some samples selected from the atlas. for the case $\lambda = +1$, (Fig. 4), the coupler curves are symmetric about the centerline, which is a straight line with $x = L_1 = \beta/2$. With different phase angle γ, different curves are generated. For the case $\lambda = -1$ (Fig. 5), with phase angle γ and $-\gamma$, the curves generated are of the same shape but mirror images. The $\pm\lambda$ curves are reflected about the centerline, $\gamma = 180°$. For the case $\lambda = \pm 2$, with phase angle $(\gamma + |2\pi/\lambda|)$, the curve generated is identical to that generated with angle γ, (Figs. 6 and 7). With angle



(a)



(b)

Fig. 4 (a) A sample with $\lambda = +1$, $0° < \gamma < 180°$ (b) A sample with $\lambda = +1$, $180° < \gamma < 360°$.

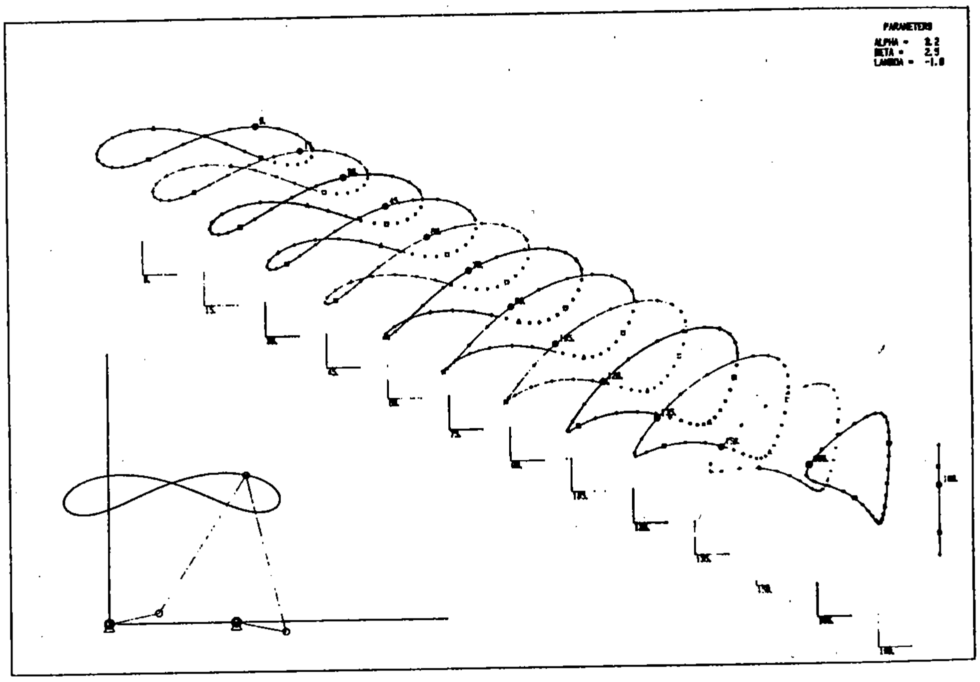


Fig. 5. A sample with $\lambda = -1$.

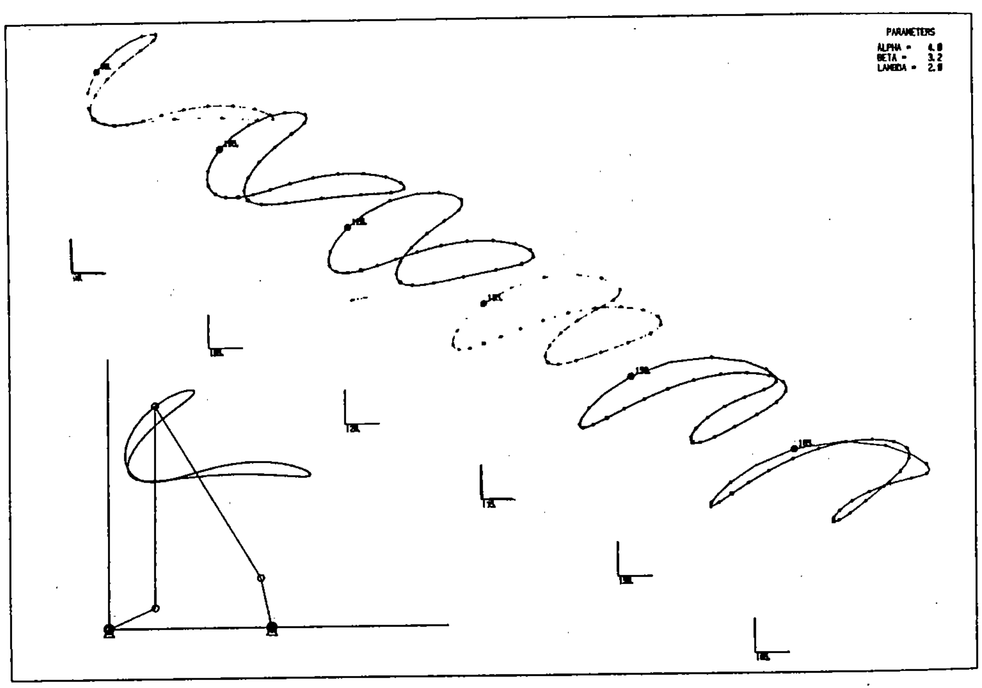


Fig. 6. A sample with $\lambda = +2$.

increment of 15°, the combinations of λ and γ used in the atlas are listed in Table 1.

Combinations of parameters α and β are determined such that the mechanism will possess mobility when $L = BD$ (Fig. 3), the condition of mobility is

$$L = BD = \sqrt{(X_D^2 - X_B^2) + (Y_D^2 - Y_B^2)} \leq 2\alpha$$

The combinations of α and β used in the atlas are listed in Table 2.

Table 1. Combinations of λ and γ

λ	RANGE OF γ	INCREMENT	NUMBER OF CURVES
+1	0 - 360	15	24
-1	0 - 180	15	12
±2	0 - 180	15	12

A scale diagram of the linkage is shown, and the three basic parameters α, β and λ are listed on each page. The value of phase angle is labeled near the first point of each curve. A small circle is drawn at each node on the curve. For those nodes which corrrespond to rotation angle $\theta = 60\,\text{n}$ ($\text{n} = 0, \ldots, 5$),

Table 2. Combinations of α and β

β \ α	2.5	3.2	4.0	5.0
2.5	•	•	•	•
3.2		•	•	•
4.0		•	•	•
5.0			•	•
6.4				•

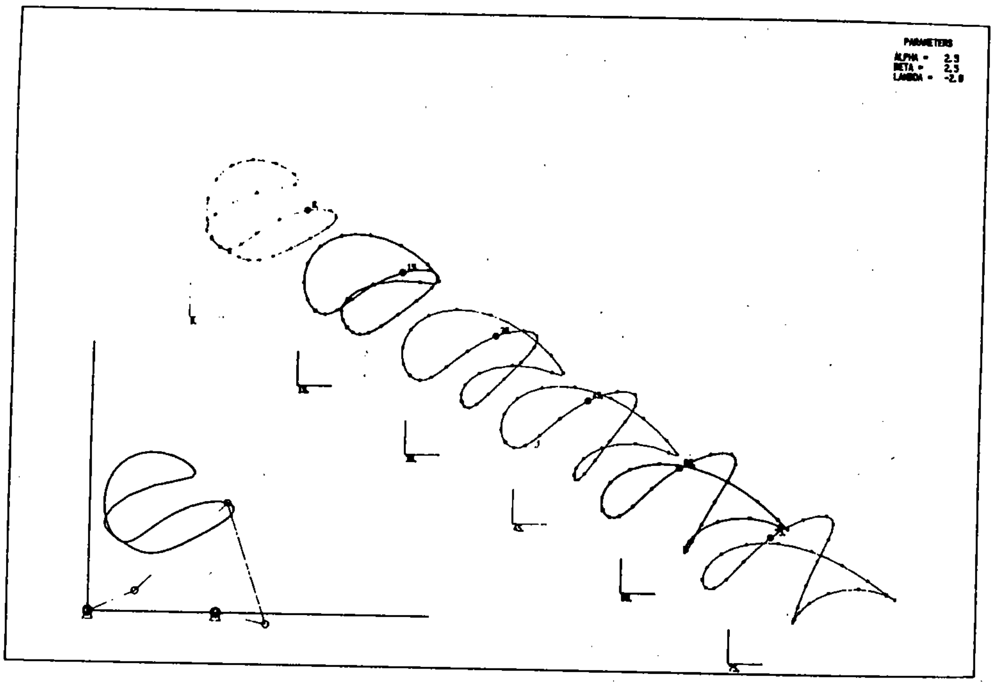


Fig. 7. A sample with $\lambda = -2$.

symbols of larger size and different shape, such as a triangle or a square, are used. By these symbols, a user can identify the starting point and see the distribution of velocity along the path, for example, when seeking a path with constant velocity or a path with fast return. The atlas is arranged by first varying the gear ratio λ, because the curves generated with different gear ratio possess entirely different shape and nature. In a group of curves with the same gear ratio, parameters α and β are varied in increasing order.

3. SUGGESTIONS ON PRACTICAL APPLICATION

Besides those applications suggested by previous researchers, the following are also interesting and should be given further investigation:

(1) Certain regions of some curves are very close to a straight line, and the velocity on the straight line part does not vary very much. Straight line motions both with and without a constant velocity profile are very useful in industrial devices.

(2) On some curves the velocity varies markedly along the path and the motion has a fast return.

(3) Non-unity gear ratio GFBM's will give multi-loop coupler curves which can be used for multiple frequency outputs (i.e. double strike per input revolution or multiple dwell mechanisms[5]).

The GFBM can be applied everywhere the four-bar path-generator[16] is used, and a design with more tailored performance can be obtained with it due to its more numerous parameters. It is the authors' opinion that GFBM have not been used as widely in machine design as they perhaps should be used. The purpose of this atlas is then to serve as a resource to machine designers to aid in the design and selection of potential GFBM for specific applications.

4. OPTIMUM DESIGN

The design of mechanisms can be formulated as a problem in nonlinear programming, which has already become a very powerful method in mechanism design. During the last twenty years, a great amount of work has been carried out in this field[7–13].

The general mathematical model of a nonlinear programming problem can be stated in the following form:

$$\text{Minimize } F(X)$$

where (X) is the vector of the design variables subject to the inequality constraints

$$g_i(X) < 0 \quad i = 1, 2, \ldots, p$$

and the equality constraints

$$h_j(X) = 0 \quad j = 1, 2, \ldots, q.$$

Usually, in path-generating mechanism design problems, a relation exists between the rotation angle of the input crank and the position of the tracer-point. The maximum absolute value or the mean root value of the error between the desired and the actual positions is usually used as the objective function to be minimized. In the case where no such relation exists, an area criterion should be used[9], where the area between the desired and the actual curves is minimized.

For the geared five-bar linkage (GFBM), the design variable vector (X) contains the following 12 independent variables (Fig. 8):

L_1, L_2, L_3, L_4–length of the links.

X_A, Y_A, X_E, Y_E–coordinates of the two fixed pivots.

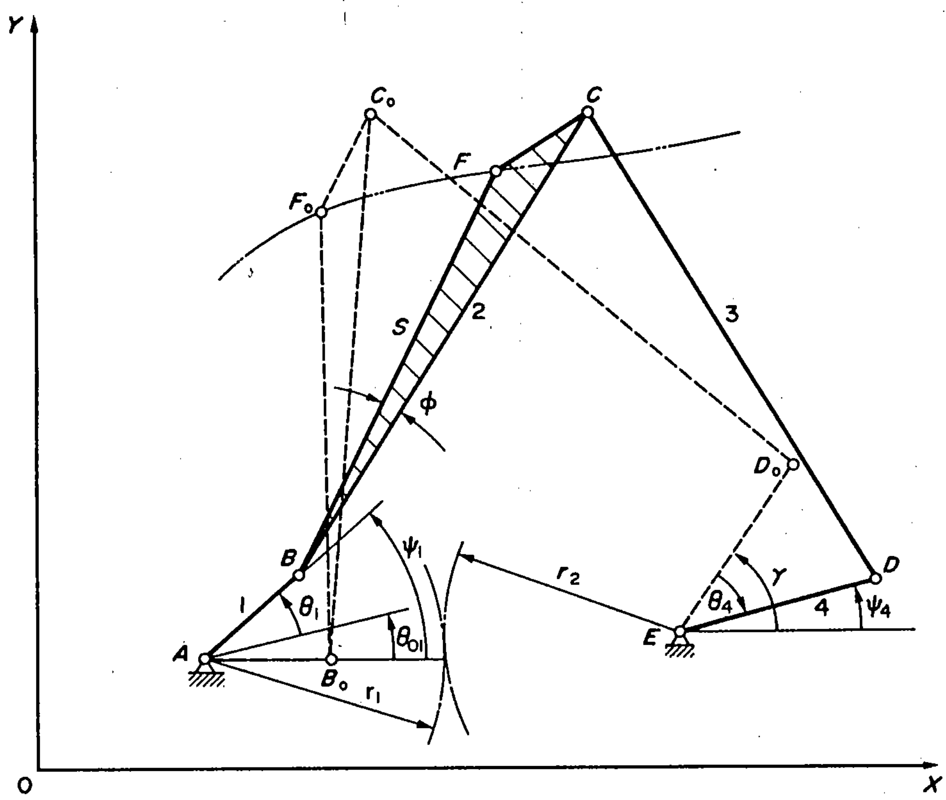


Fig. 8. Geometry of general type of GFBM.

S and ϕ–position parameters of the tracer-point F on the link 2 (or link 3).

θ_{01}–the angle between X-axis of the fixed frame and the driving crank angle reference position. Actually, θ_{01} is not a geometrical parameter. However, for a particular design problem where a specified relationship exists between the coordinates of the tracer-point and the input angle, the initial position of the crank may be arbitrary. To increase the degree-of-freedom of design, θ_{01} is taken as one of the design variables.

γ–the phase angle, namely the angle between the driven crank ED and the X-axis as both θ_{01} and the rotation angle of the driving crank θ_1 are equal to zero.

Another design variable, which has not been included in the vector (X), is the gear ratio

$$\lambda = W_1/W_4 = \pm Z_4/Z_1 = \pm m/n$$

where W_1 and W_4 are the angular velocity of the two cranks, and Z_1 and Z_4 are the numbers of the teeth of the two gears. In formula (1) m and n are two primes. After the driving crank has rotated m cycles, the linkage completes a motion period. If γ is not a whole number, the curve generated will be very complex. Usually, only some simple whole numbers, for example, ± 1, ± 2 and ± 3 are used.

A variety of numerical techniques have been developed to search for the minimum point[14, 15]. Up to now, however, there is no known method for locating the global minimum directly. The main difficulty in the optimum design of any path-generating mechanism is that the objective function of the path errors has many local minimum points. In the design of a path-generating mechanism, many linkages with a local minimum value of the objective function are obtained when different starting points are used. The achievement of a better design depends on how good the starting design is. If the starting design is far from the global minimum point, the designer often obtains a local minimum point which is sometimes entirely inapplicable. Even in random search techniques, a better starting design is helpful. Experience has shown that a starting design selected from an atlas[17] usually leads to an acceptable solution, since an approximate solution can be quickly obtained by studying the sample coupler curves in the atlas.

5. COMPUTER PROGRAM

A Fortran program was written to run on a Digital Equipment DEC-20 computer. The program takes as input all of the parameters of the geared five-bar linkage as defined by the choice of an approximate solution from the atlas of coupler curves of the geared five bar linkage[17]. The design variable vectors are defined by the user. Thus the starting values for the optimization are at least reasonable, based on the selection from the atlas. The optimization algorithm is then applied and the linkage parameters are modified in the iteration. The changed parameters are reported out.

6. APPLICATION

Now, an example is given to illustrate how to perform an optimum design with the starting design selected from the atlas.

A straight line generator is to be designed. The

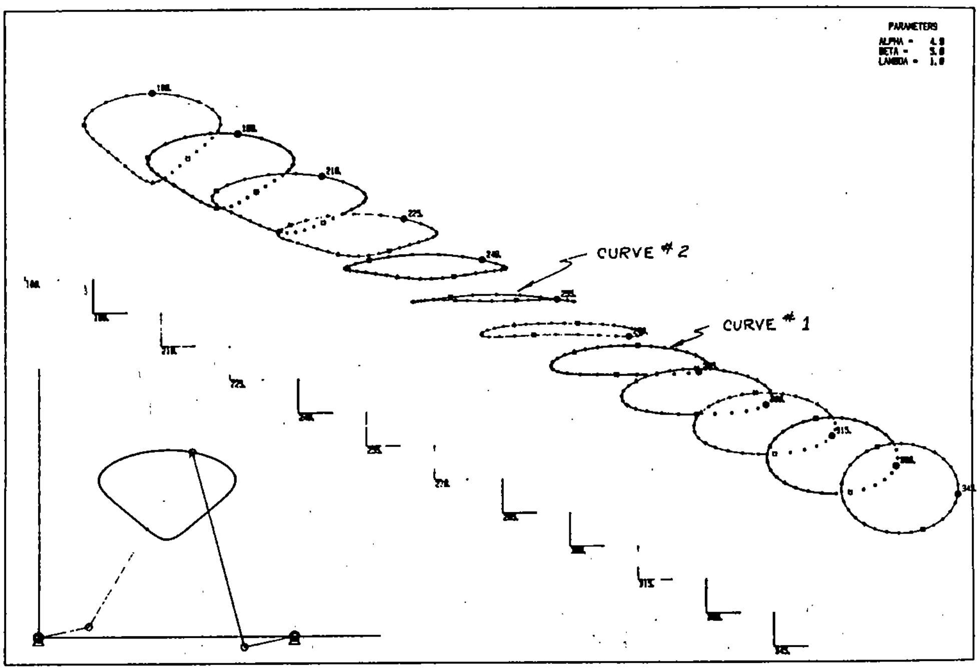


Fig. 9. A sample with $\lambda = +1$.

geared five-bar linkage is expected to generate a straight line $y_0 = 2.5$ as the input crank rotates over 150°. In the other 210° of the cycle, the tracer-point returns at a lower average velocity. The straight line should be parallel to the center-line connecting the two fixed pivots.

The maximum error

$$\delta = \max(|y - y_0|)$$

is taken as objective function to be minimized.

It is clear from the atlas that the best choice for a straight line parallel to the center-line is the mechanism with gear ratio $\gamma = 1$. In this particular case, the design variable vector is reduced to

$$X = (L_1, L_2, A, X_A, Y_A, \theta_{01}, \gamma)$$

Curve #1 in Fig. 9 is taken as the starting design:

$L_1 = 1.0$ in., $L_2 = 4.0$ in., $A = 5.0$ in.,
$X_A = 0.0$, $Y_A = 0.33$ in.,
$\gamma = 285$, $\theta_{\circ 1} = 238°$ (The point $\circ$ is taken as the first point, as shown in Fig. 9).

Generally speaking, the objective function in the optimum design of a path-generating mechanism is a multi-extremum function. To obtain a better design, a random technique is used as the coarse search method. The optimization process gives the following result:

$L_1 = 0.4716$ in., $L_2 = 3.569$ in., $A = 4.571$ in.,
$X_A = -0.265$ in., $Y_A = -0.178$ in.,
$\gamma = 264°$, $\theta_{\circ 1} = 250.2.°$

The mechanism can trace out a straight line with

$$X \min = 1.307 \text{ in.}, X \max = 2.756 \text{ in.}$$

The travel length of the tracer-point

$$L = X \max - X \min = 1.499 \text{ in.}$$

The maximum absolute error

$$\delta = 0.0042 \text{ in.}$$

The relative error

$$\Delta = \delta / L = 0.0029, \text{ or } 0.3\%$$

If the travel does not satisfy the requirement, all the sizes can be changed proportionally.

If the straight line is desired to correspond to a rotation angle of 210° and have a faster return, the curve #2 in Fig. 9 can be taken as the starting design. It is very interesting that two very different straight-line generators will be obtained beginning with nearly adjacent similar designs from the same atlas set. Note that each of the two designs provides a similar path position function but the velocities are markedly different in each case, one being a "quick return" and the other being a "slow return" mechanism, due to the velocity pattern differences between curves 1 and 2 which differ only in phase angle. It would be very difficult to find each of these local minima by random search and optimization techniques. The existence of an atlas to provide sensible starting designs for optimization is then very useful.

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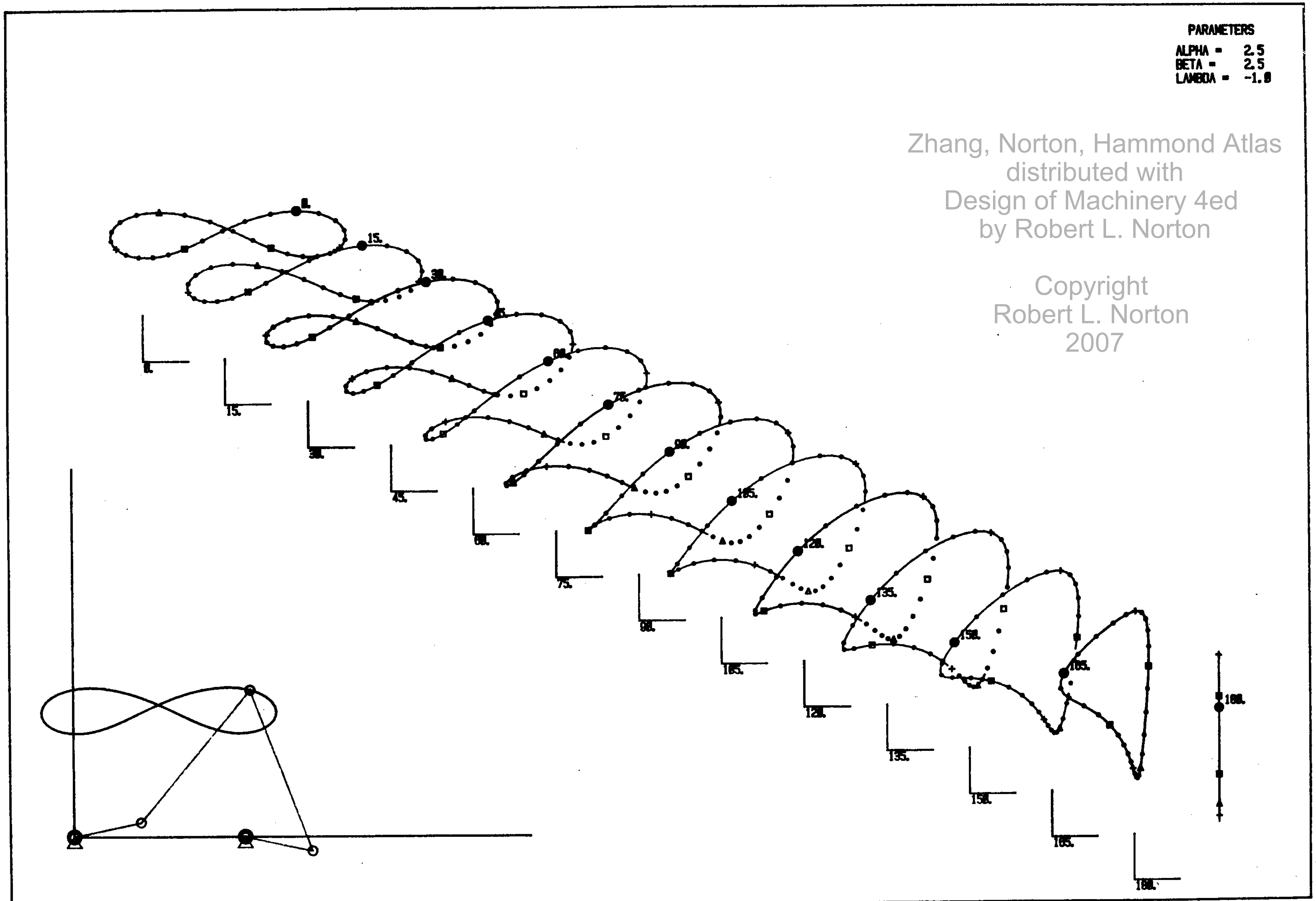
L'OPTIMISATION DES PARAMETRES D'UN MECANISME PENTALATERAL AUX ENGRENAGES POUR LA GENERATION DE TRAJECTOIRES UTILISANT UN ATLAS DE COURBES.

Résumé—Les auteurs présentent un atlas de trajectoires tracées par un point relié à un organe d'un mécanisme pentalatéral symétrique commandé par engrenages. Les rapports de démultiplication utilisés sont $+1$, -1, $+2$ et -2. Les résultats sont présentés en six ou douze courbes par page, arrangés en projection axonométrique pour illustrer l'enveloppe de la famille des courbes qu'on obtient en faisant varier la phase entre les engrenages. Des points sur les courbes représentent les déplacements angulaires équidistants des manivelles motrices et fournissent ainsi l'information sur la vitesse du point sur la trajectoire.

Les courbes originales sont tracées en quatre couleurs sur des feuilles de 11×17 pouces. Ensuite, des algorithmes d'optimisation sont utilisés pour obtenir la trajectoire désirée avec un minimum de déviation. Comme exemple on utilise le mécanisme pentalatéral pour générer une droite avec une erreur de position de 1%.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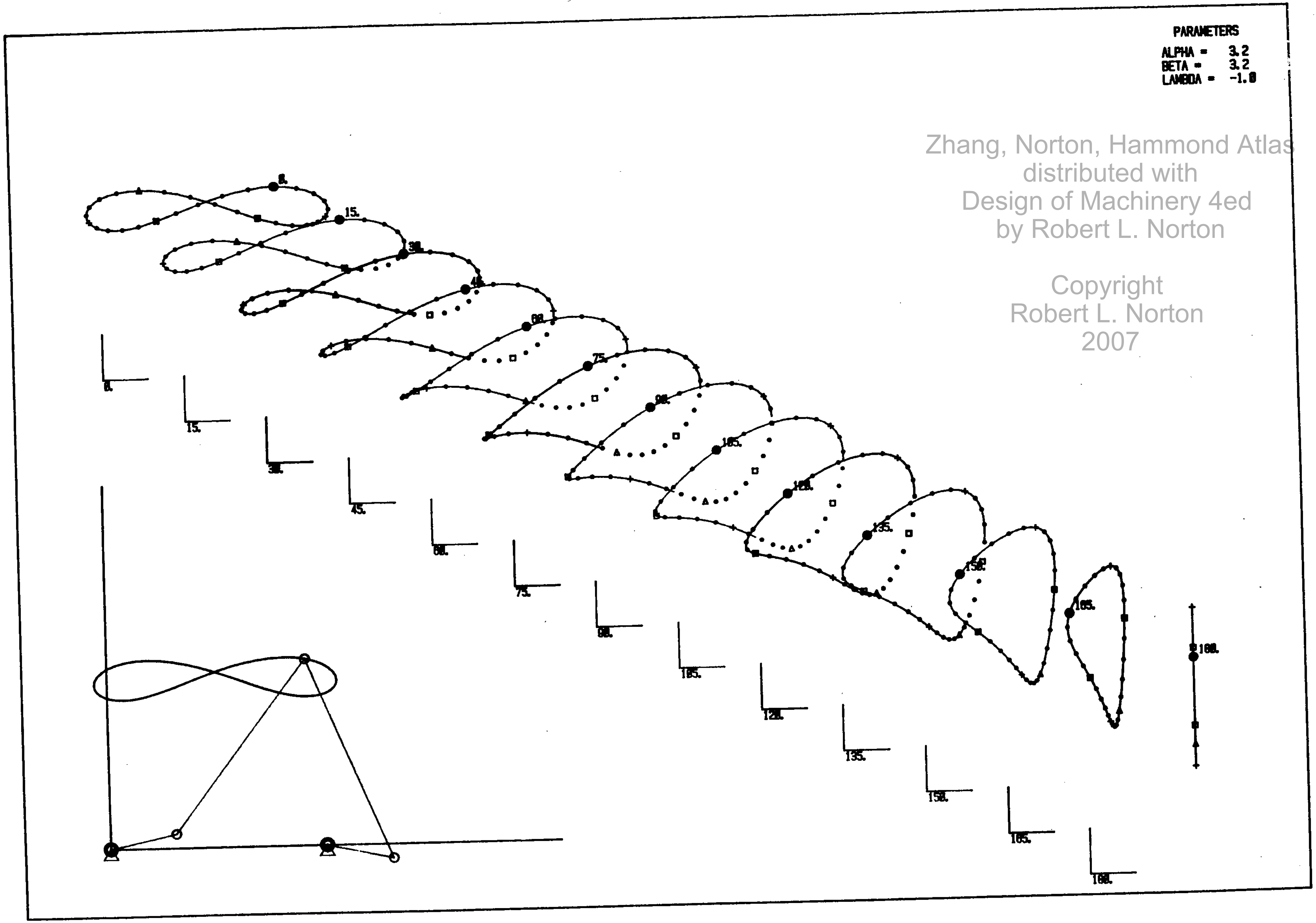
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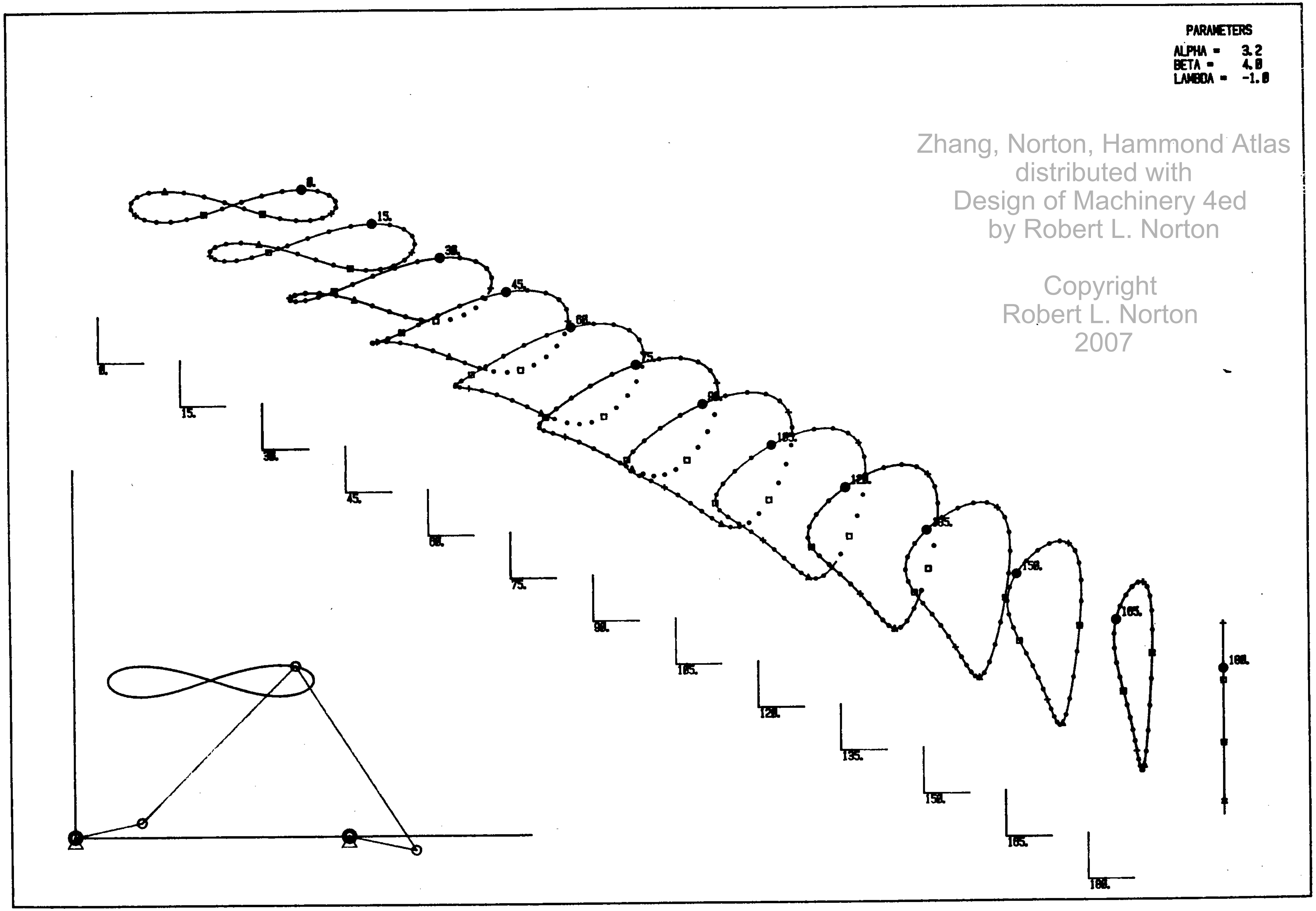
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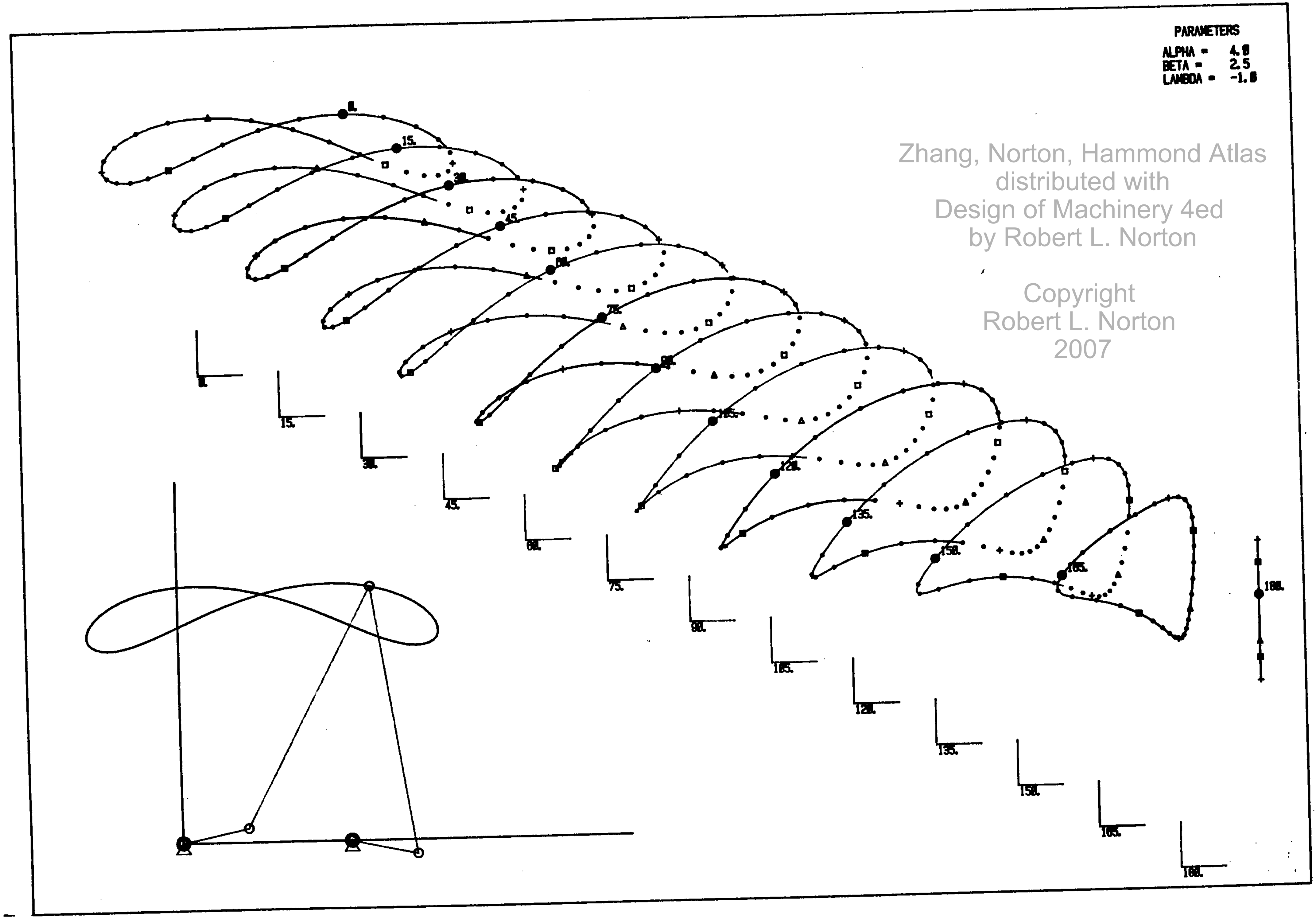
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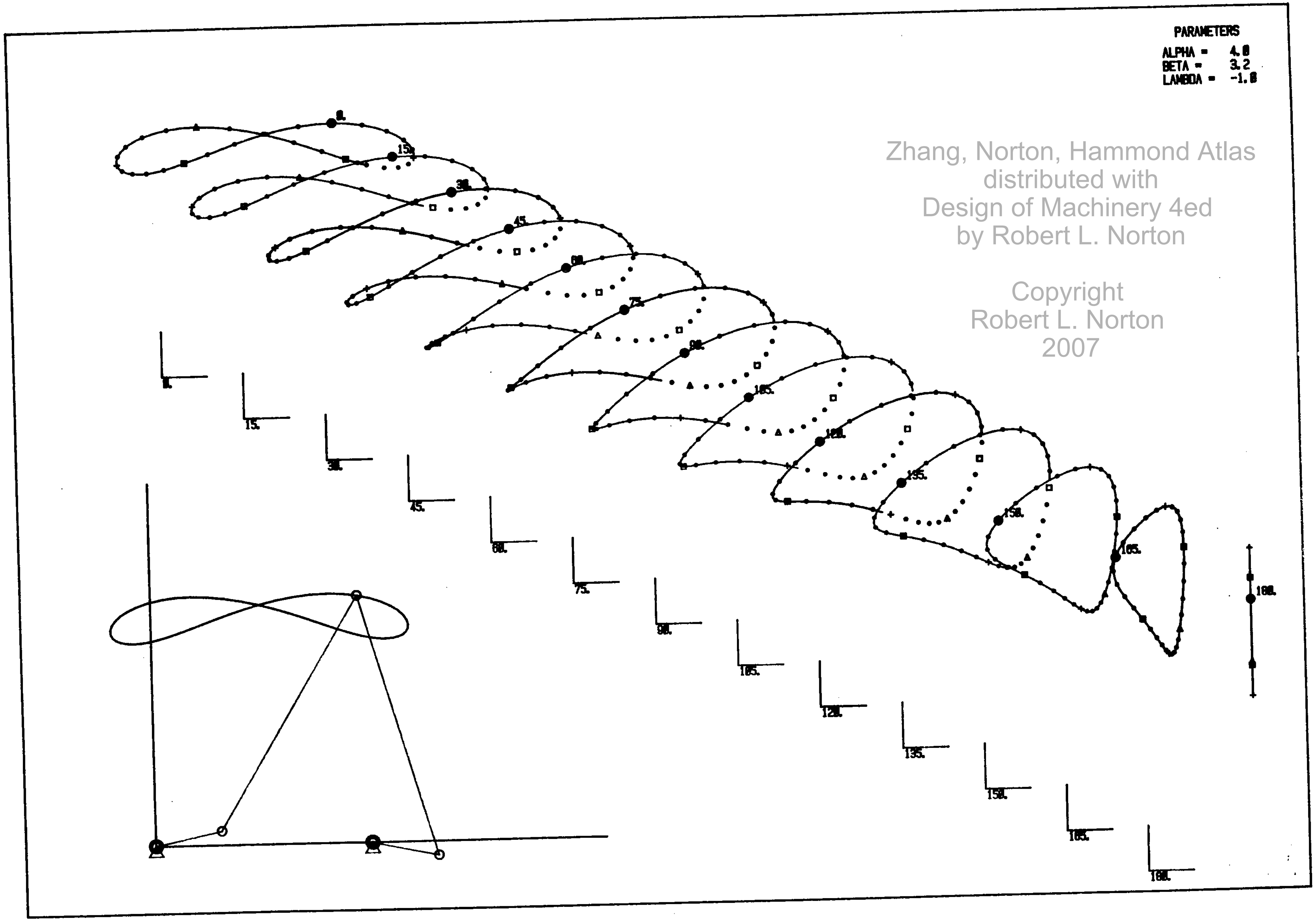
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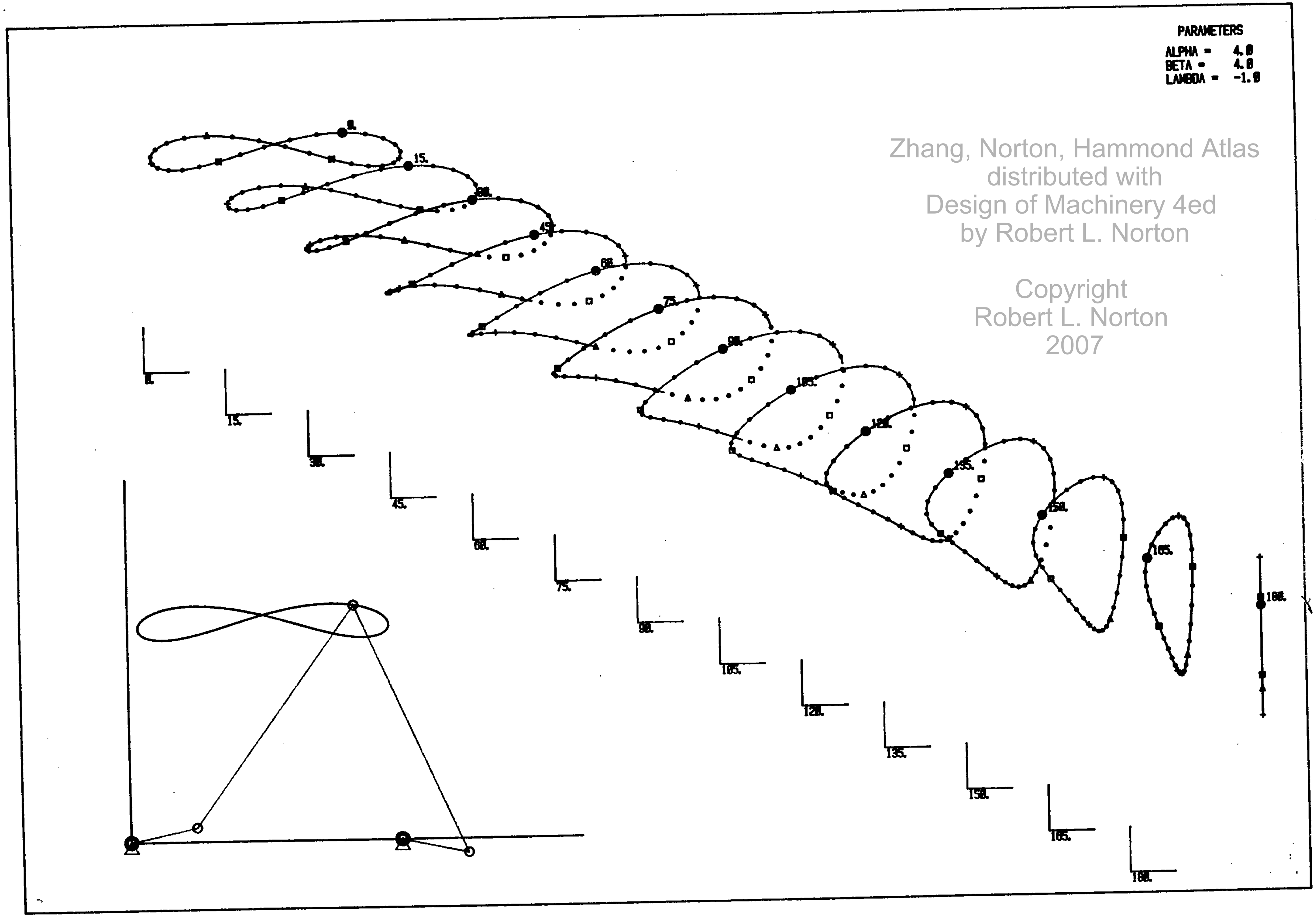

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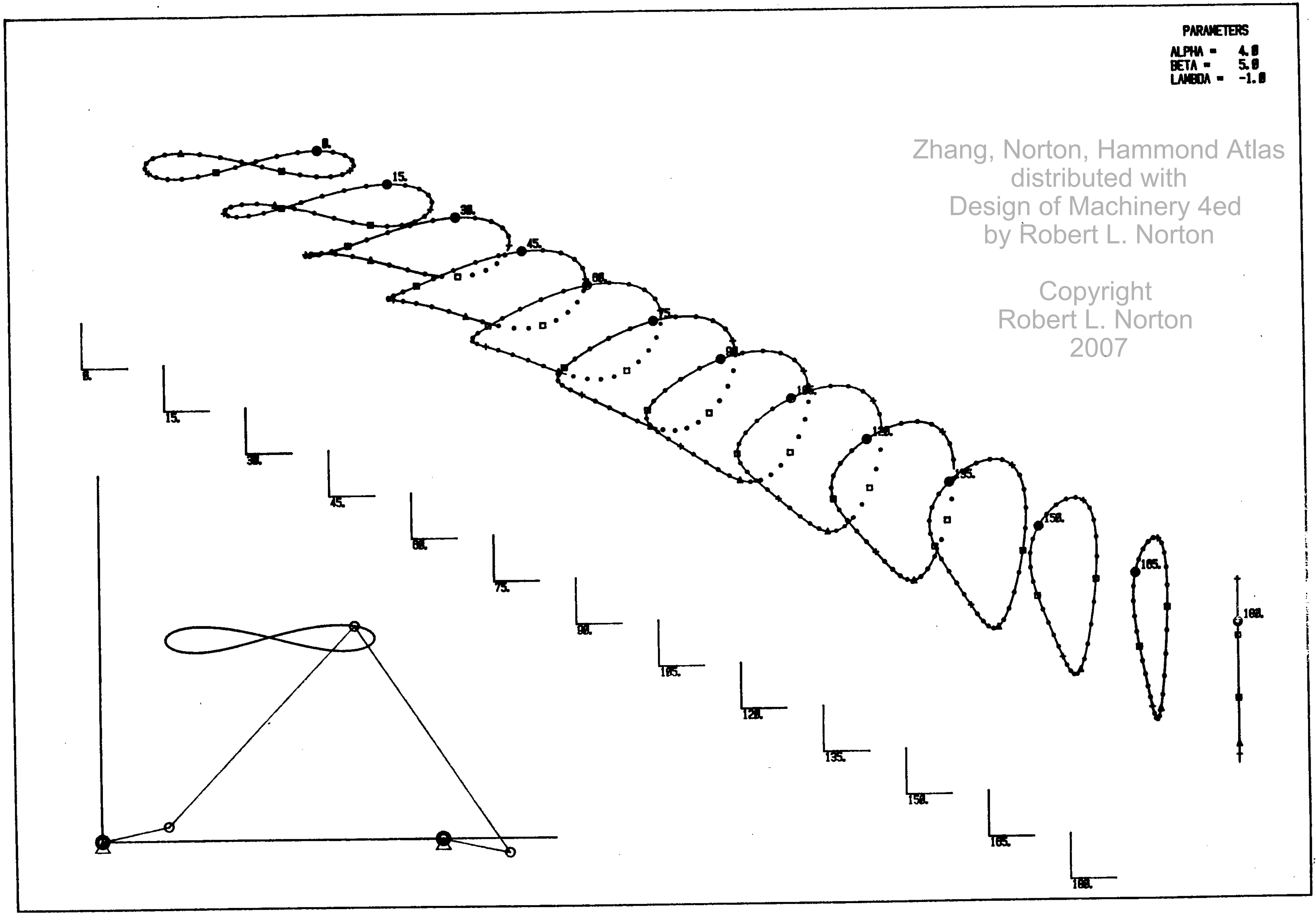
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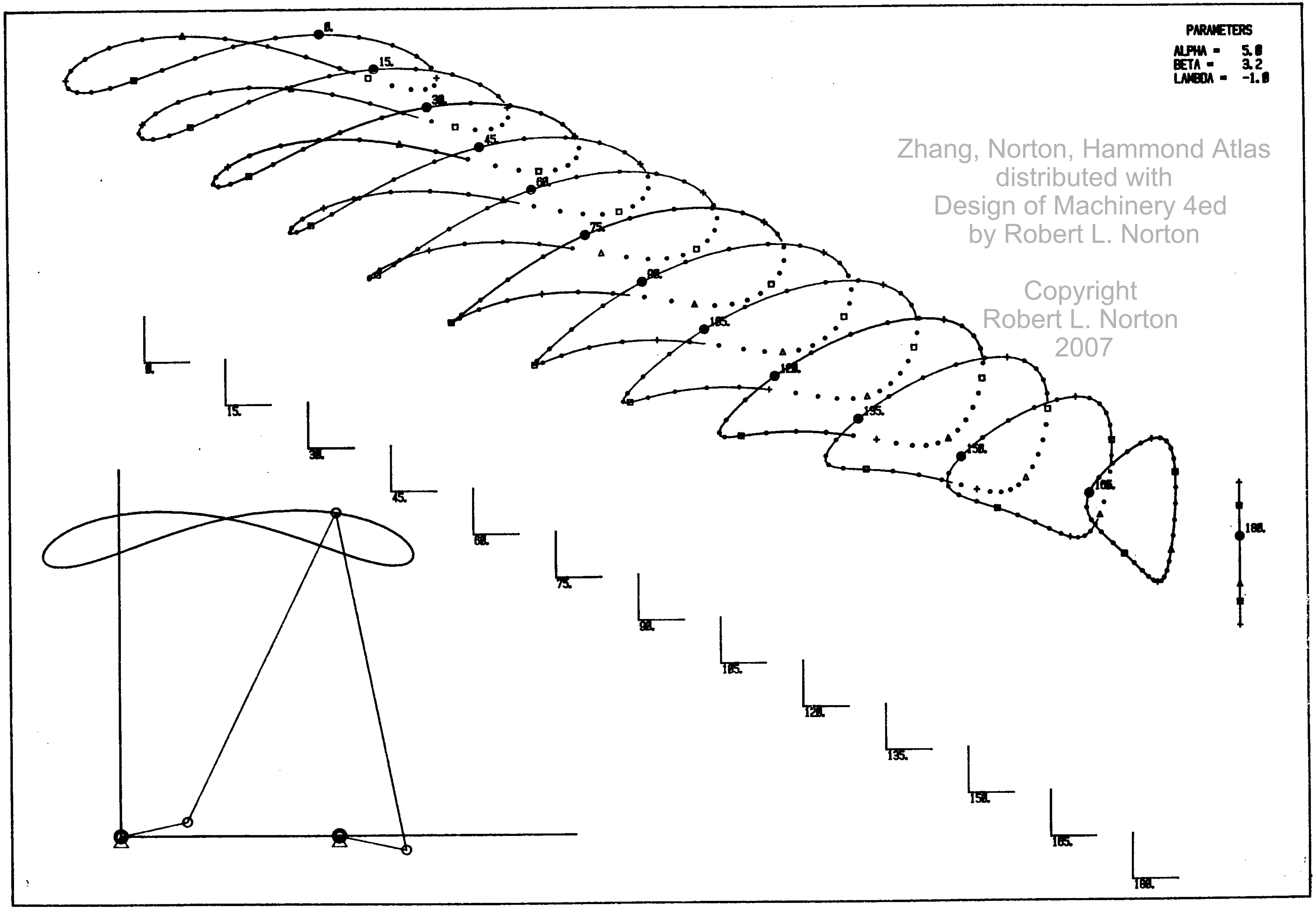
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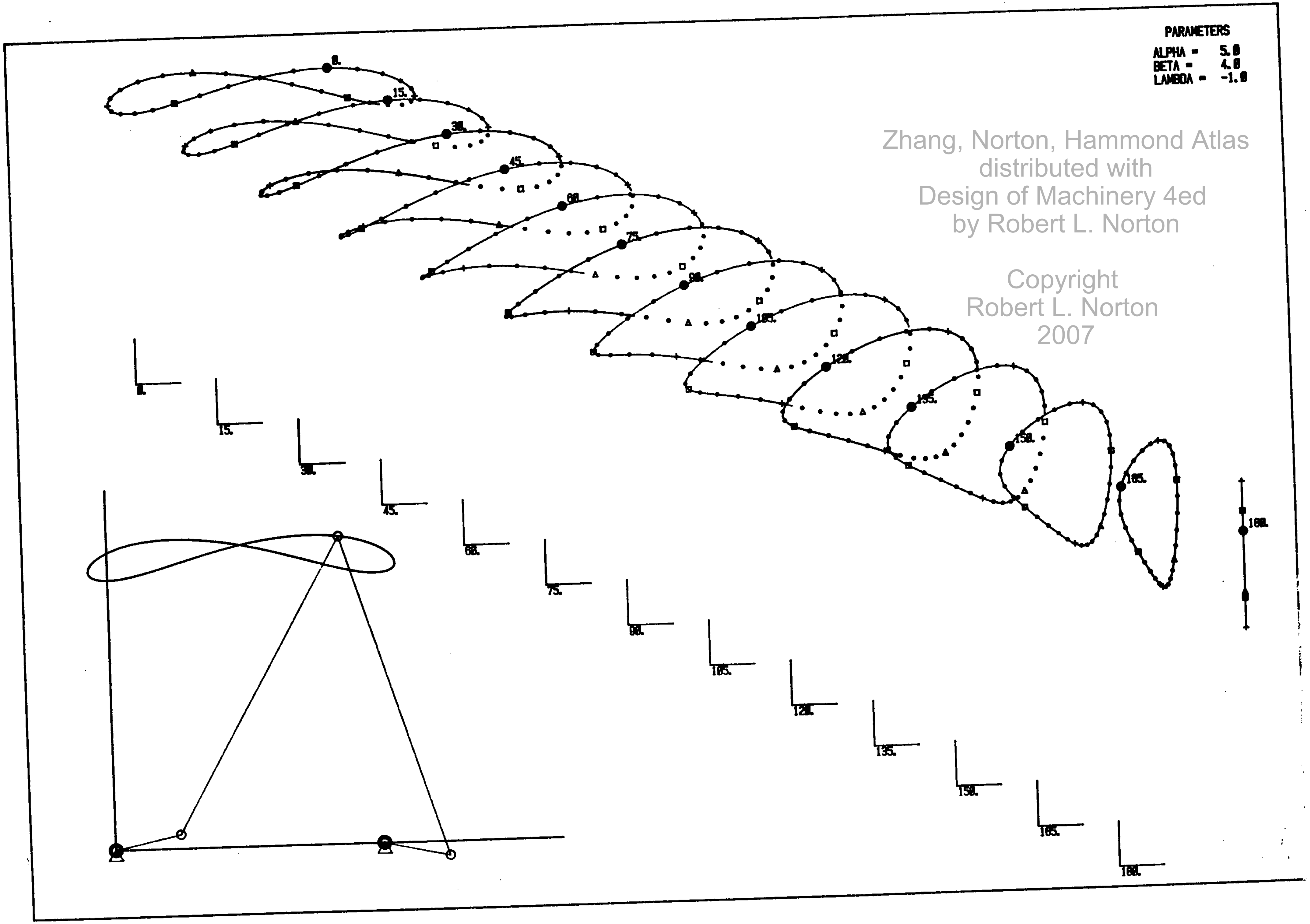
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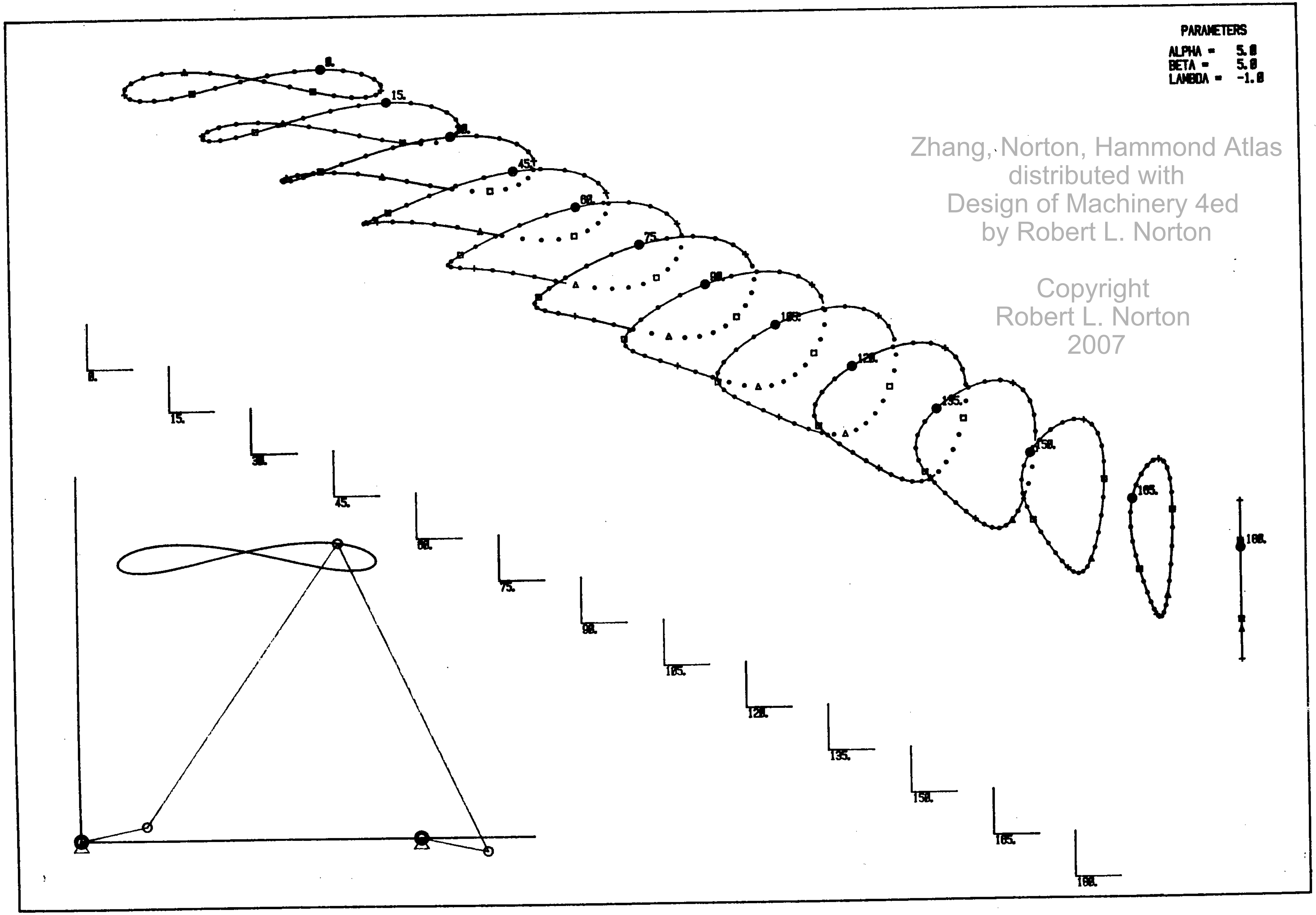
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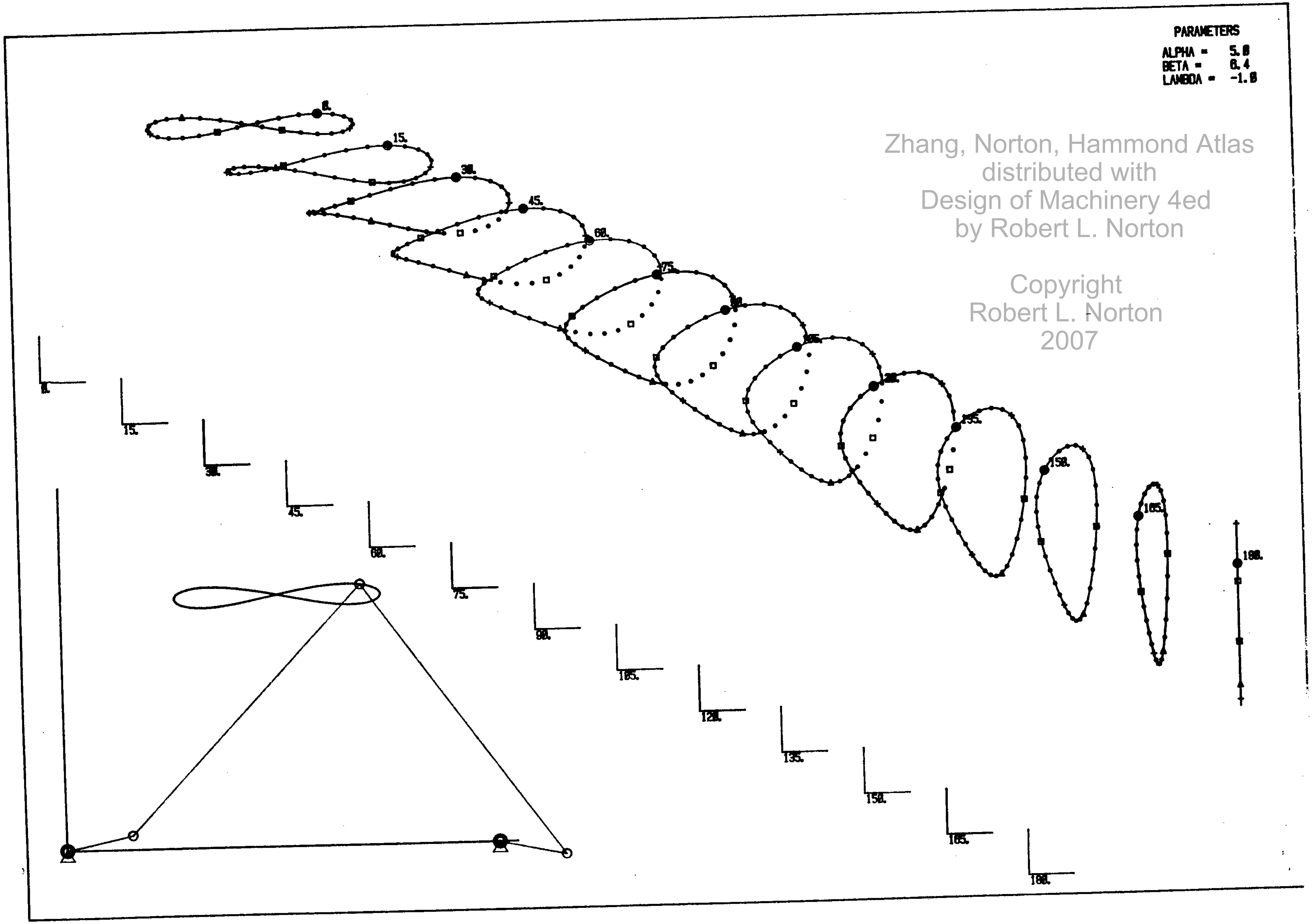
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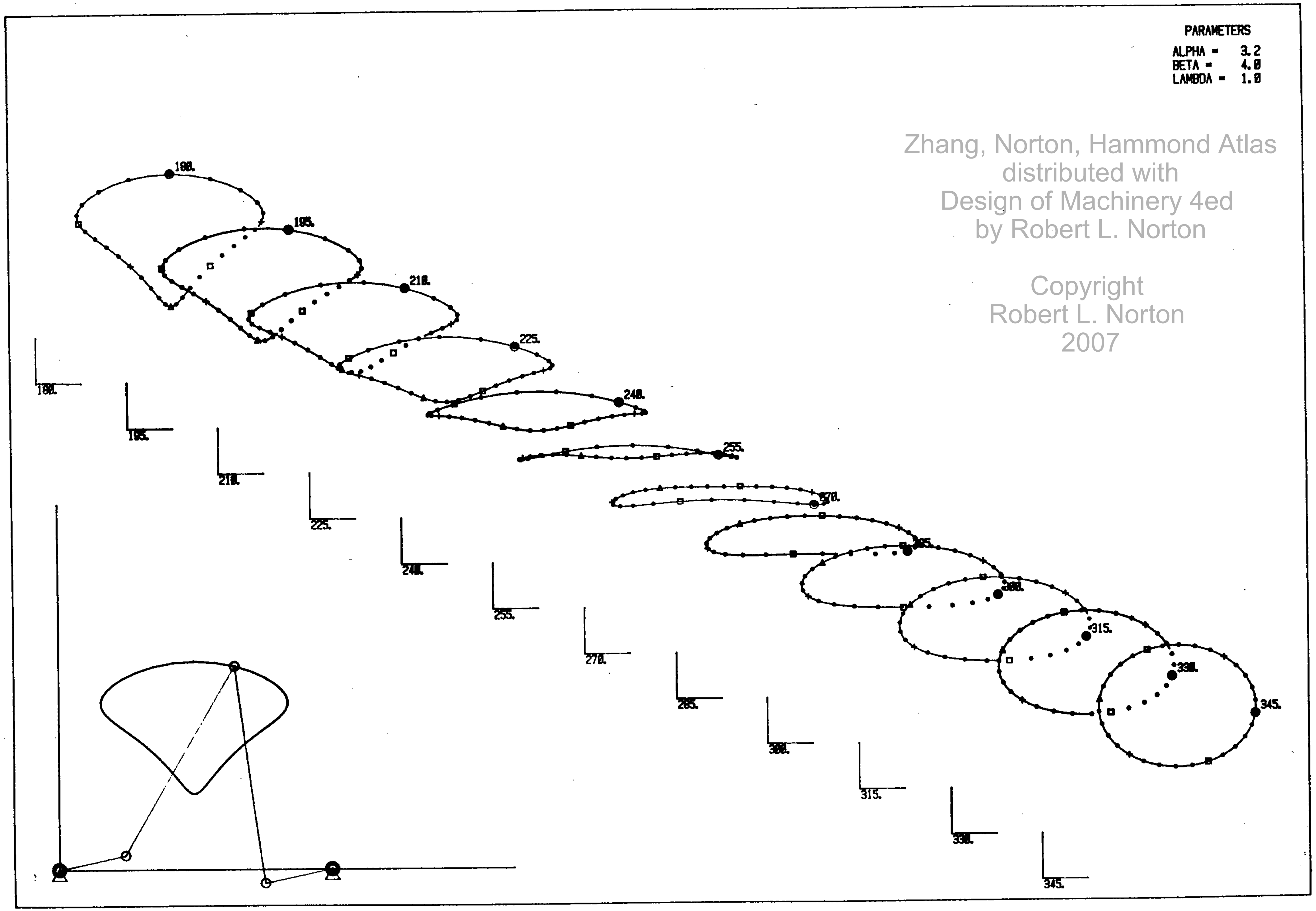
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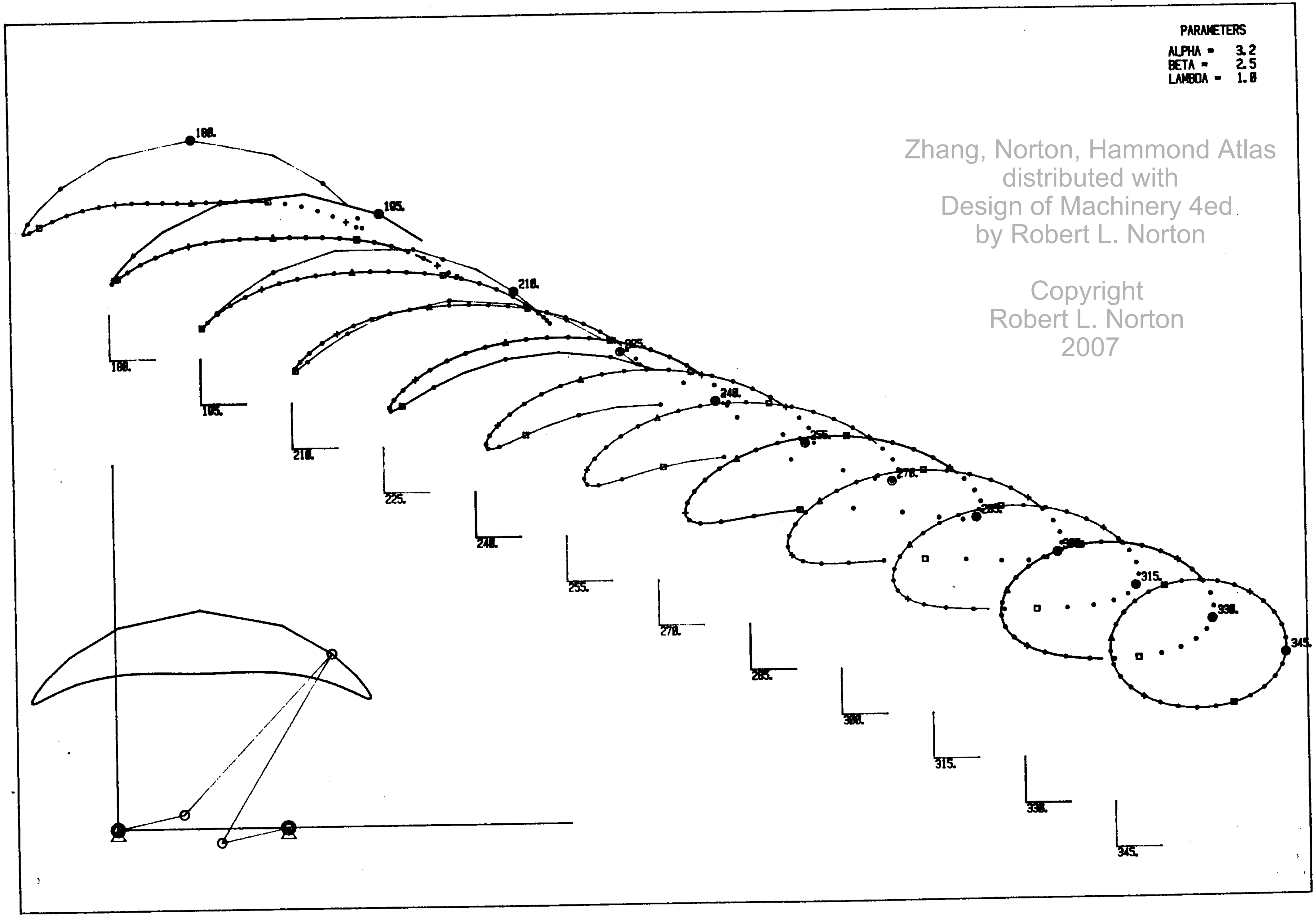
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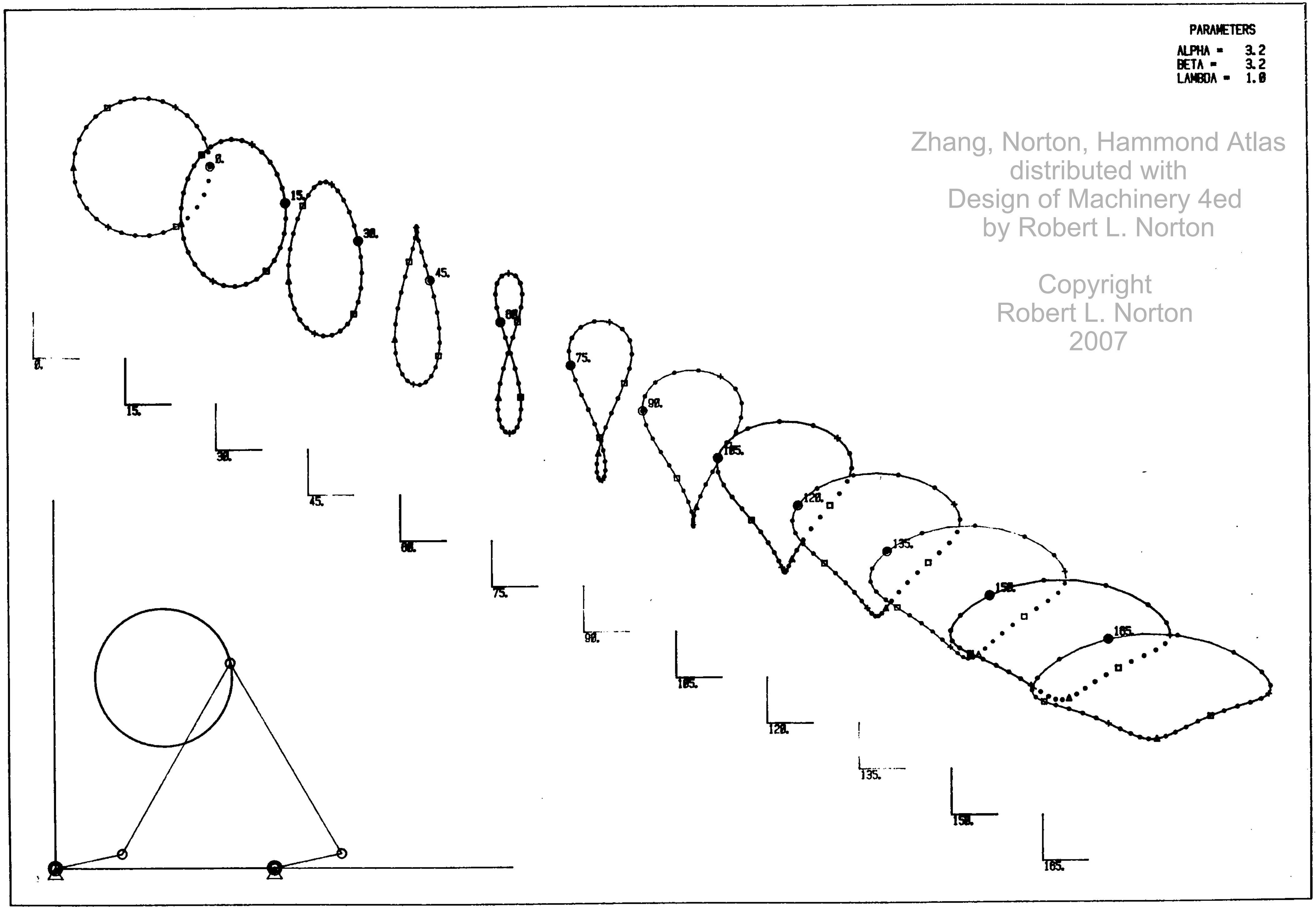
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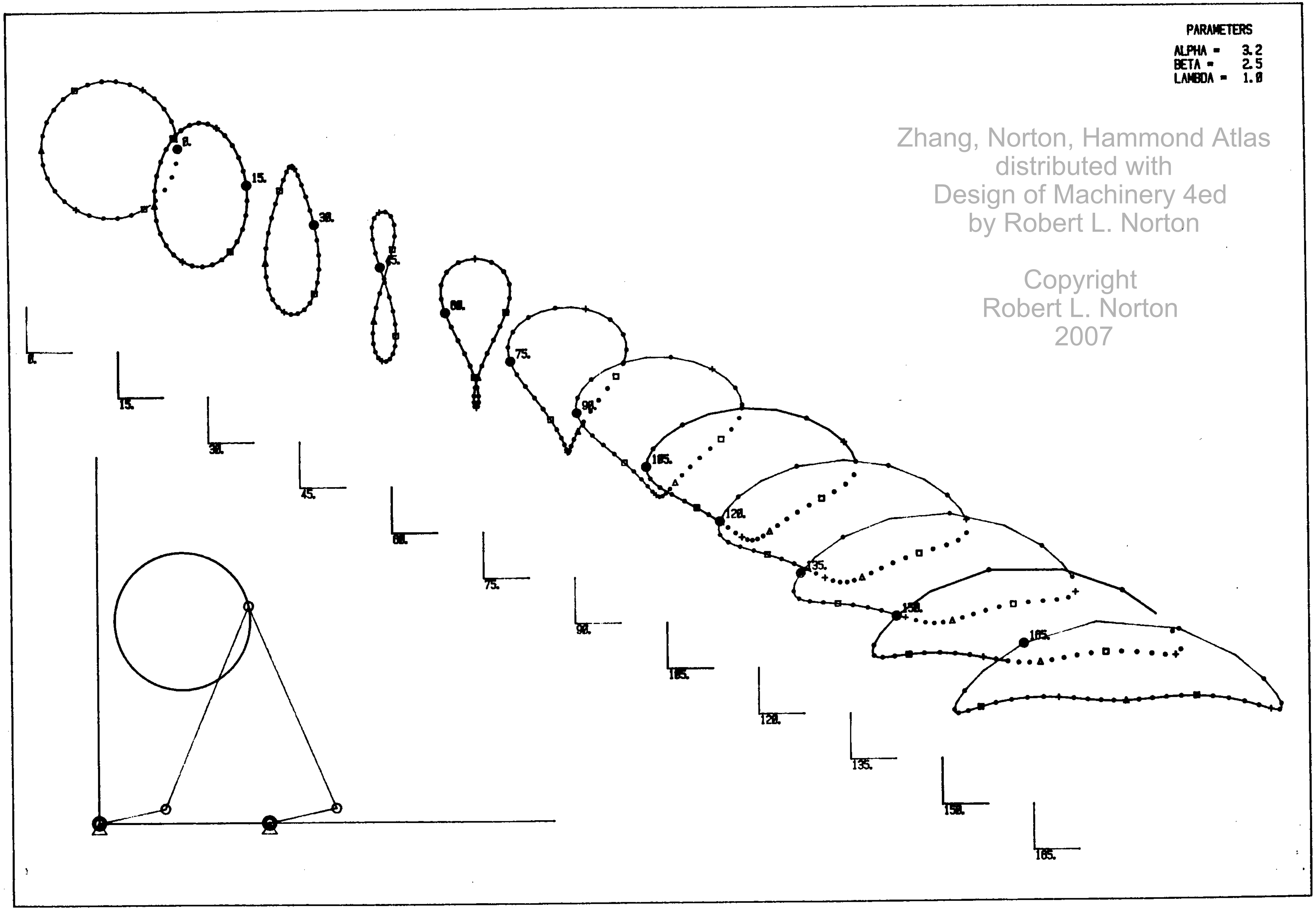

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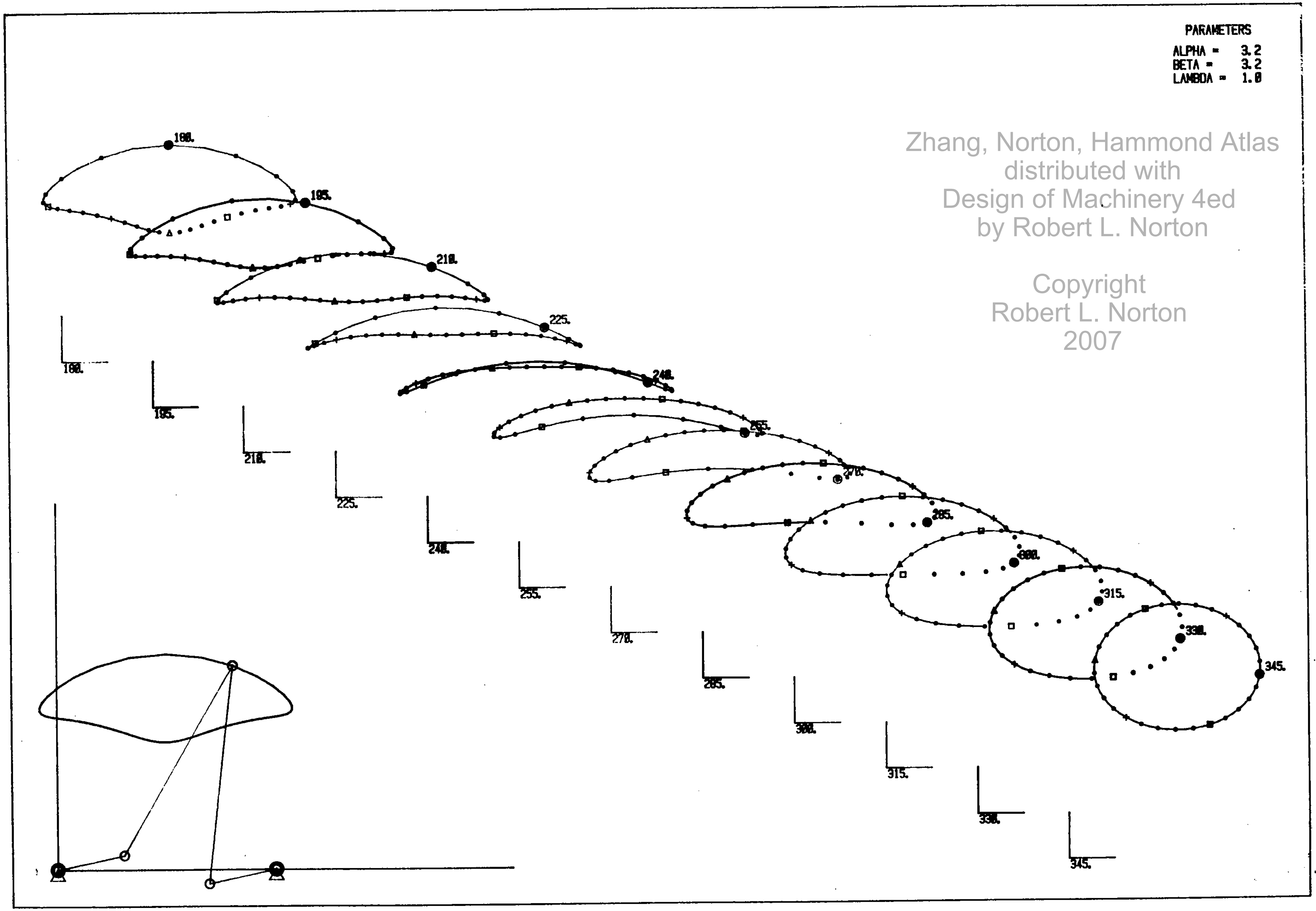
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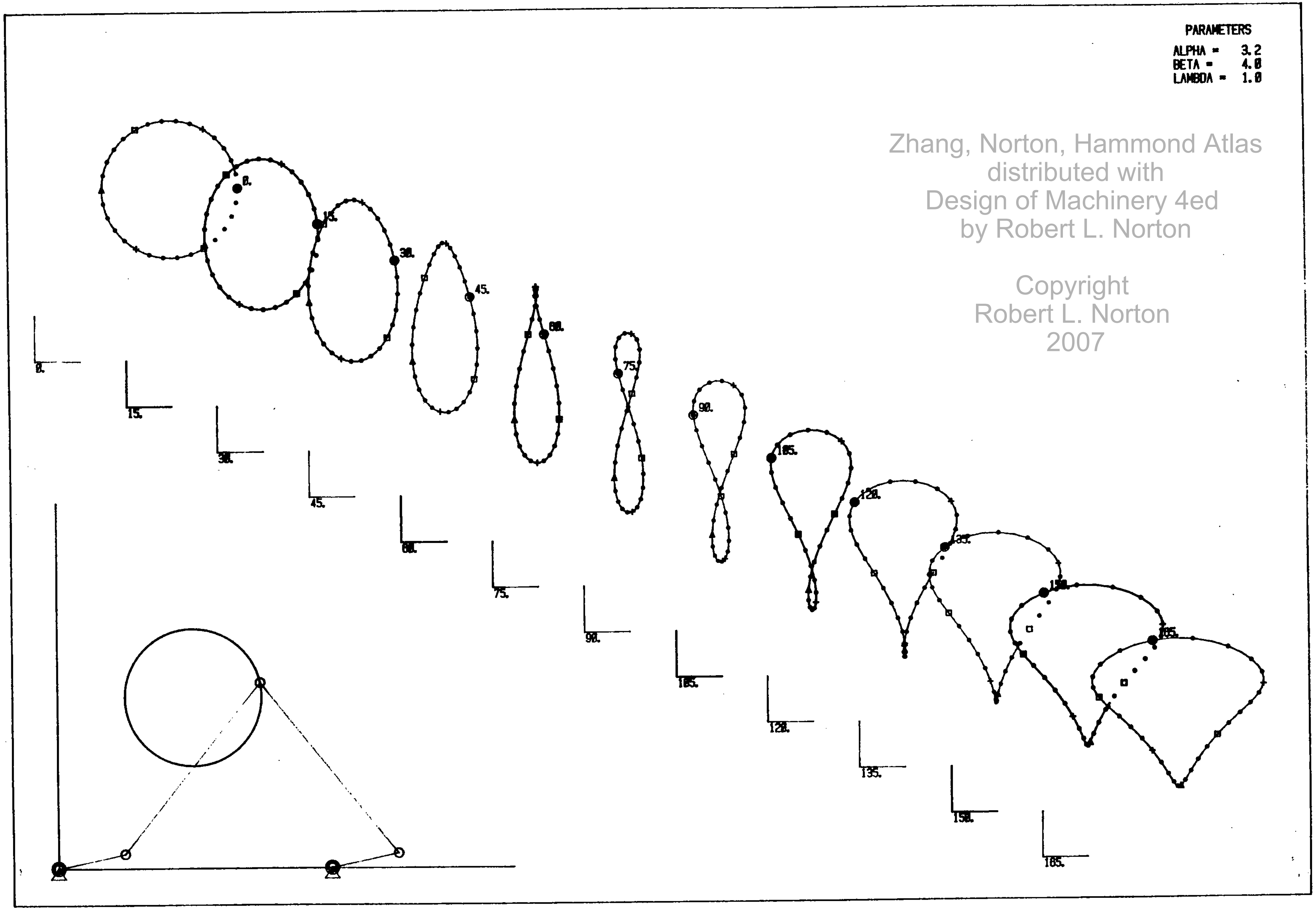
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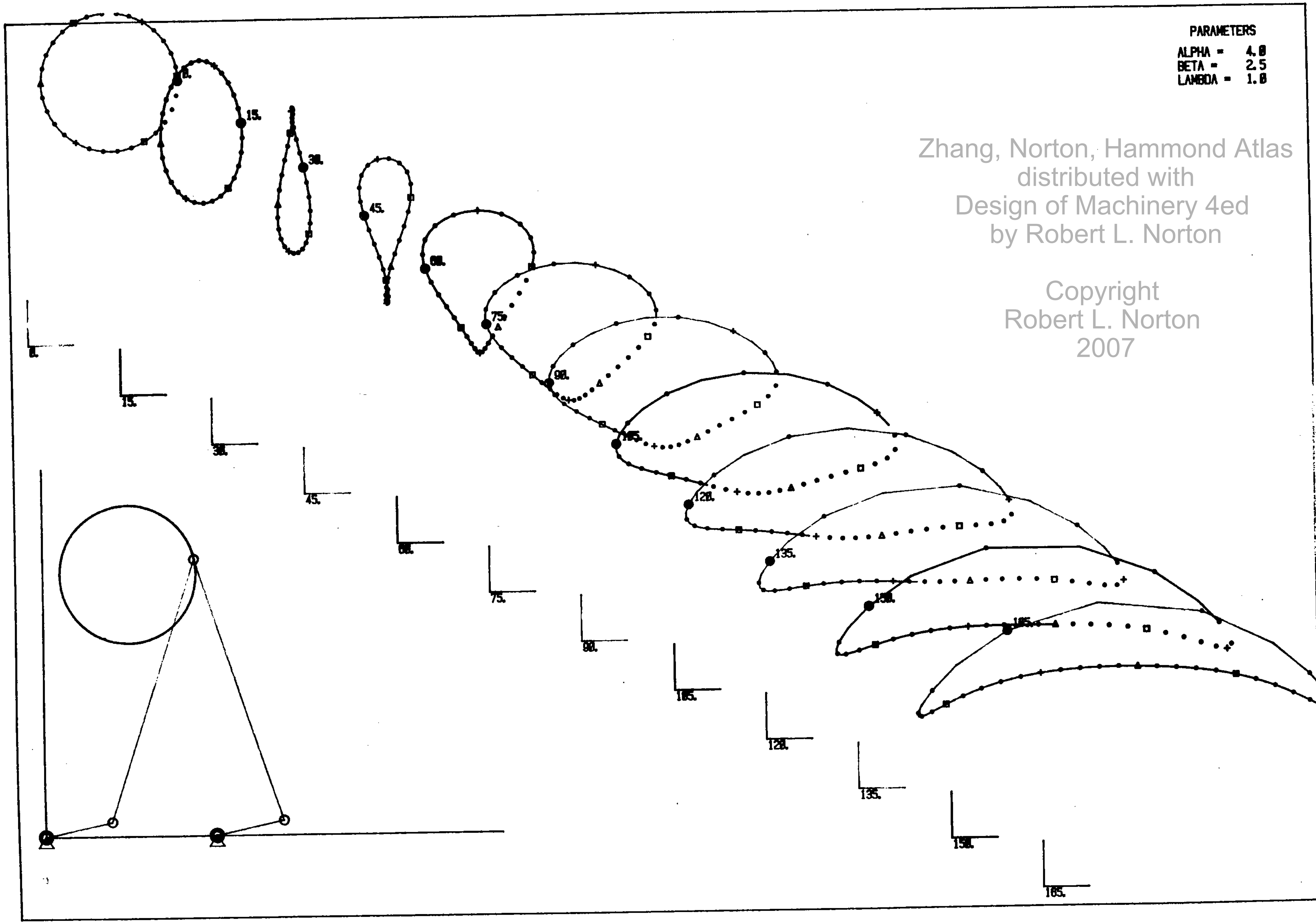
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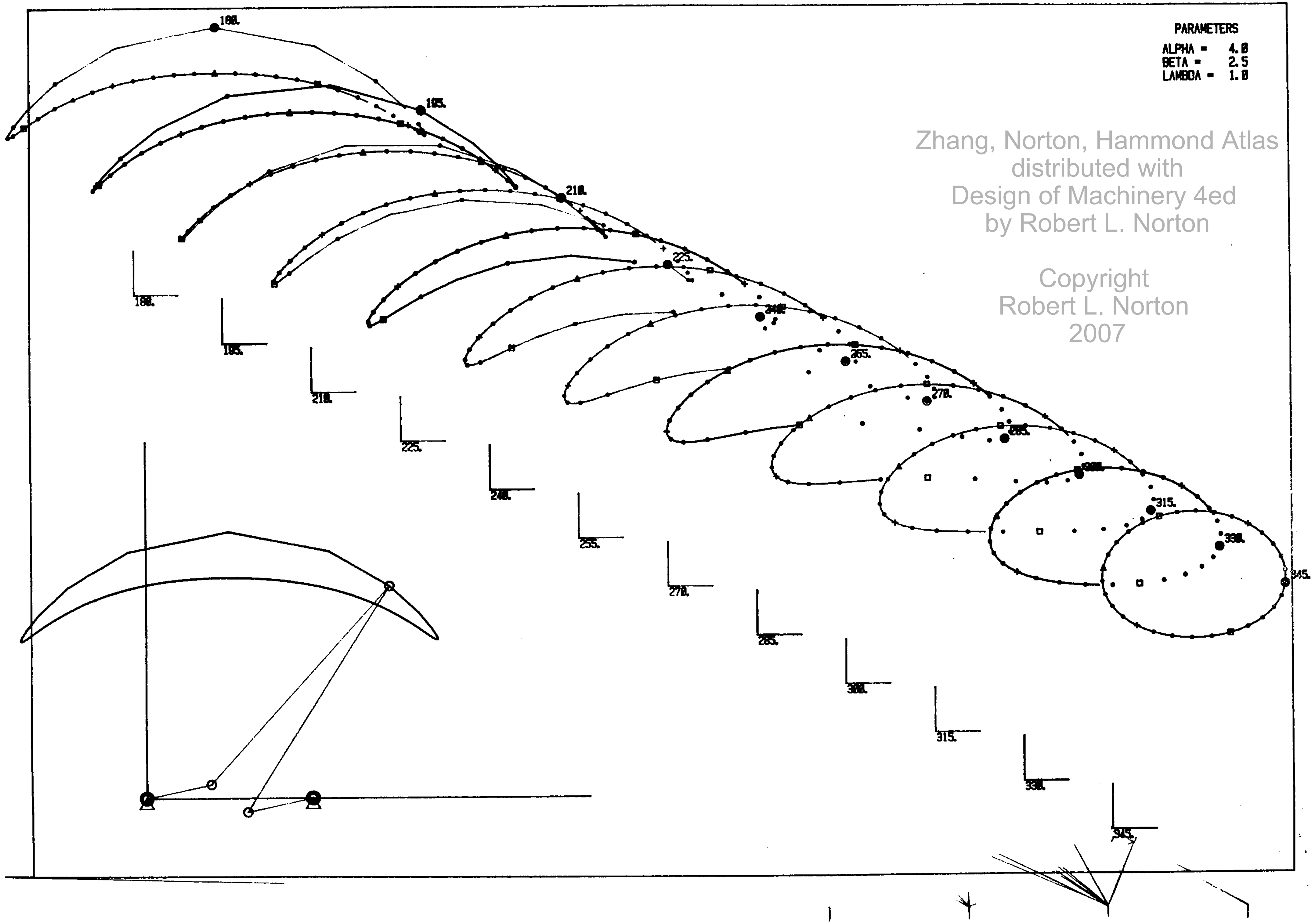

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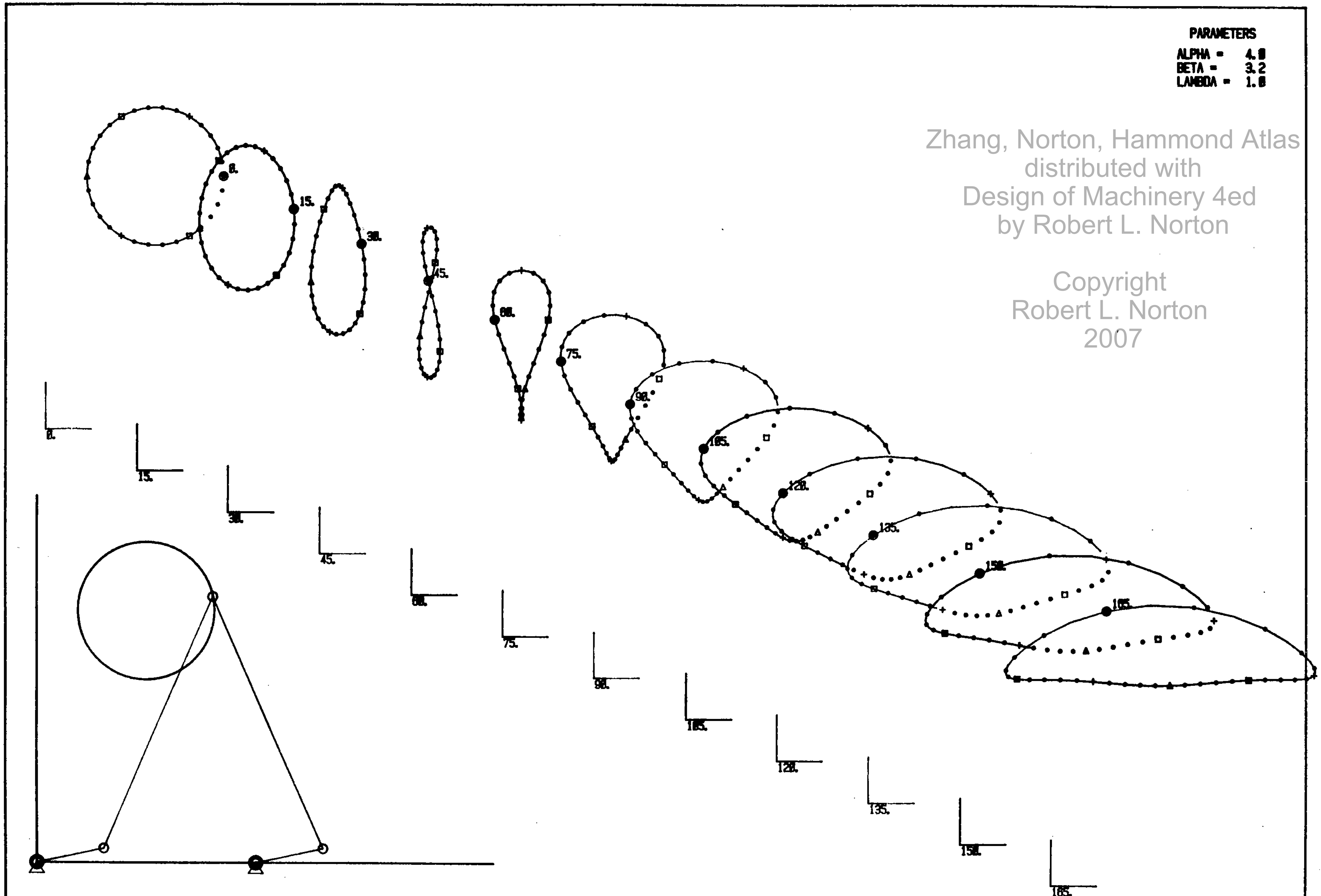

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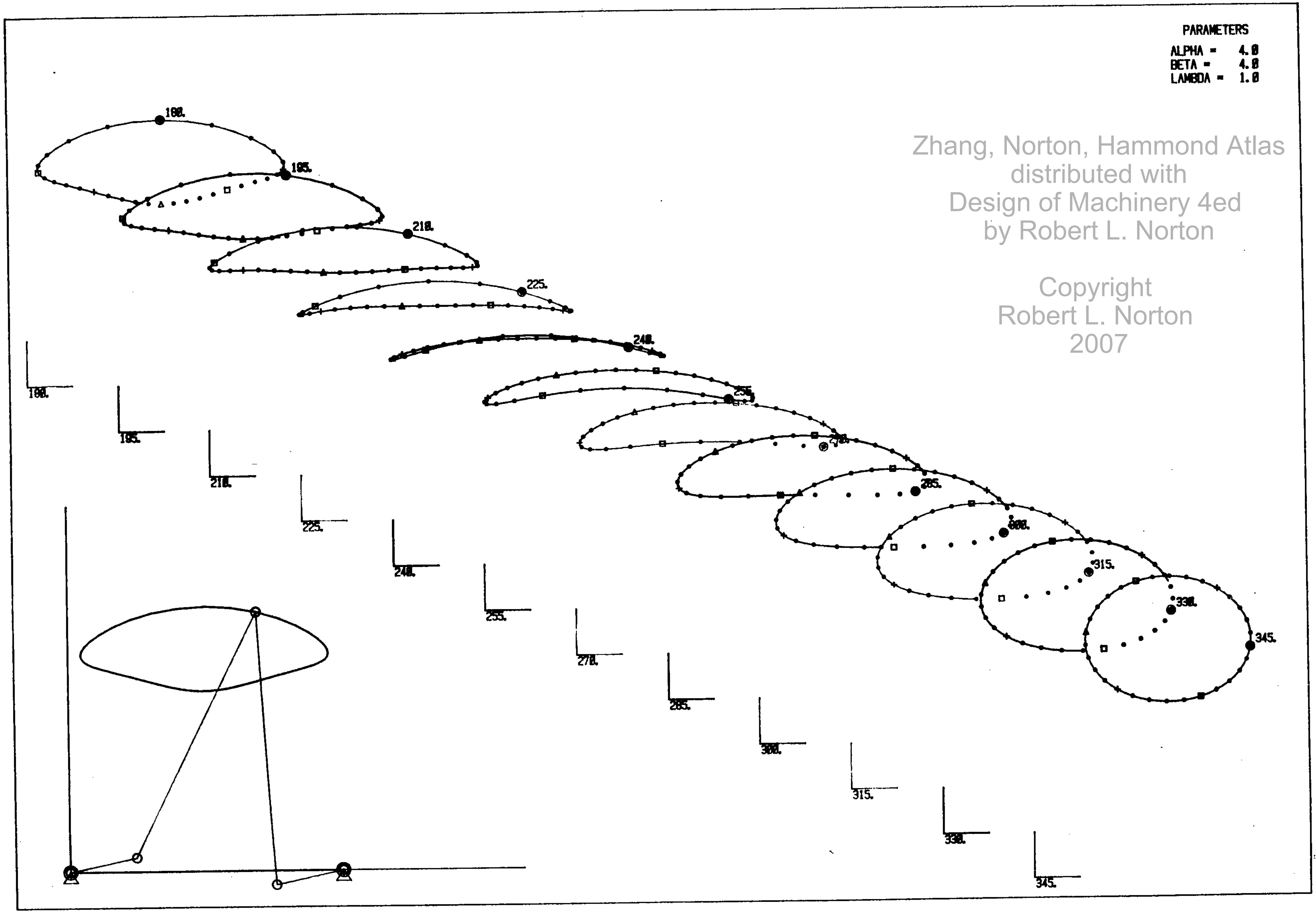
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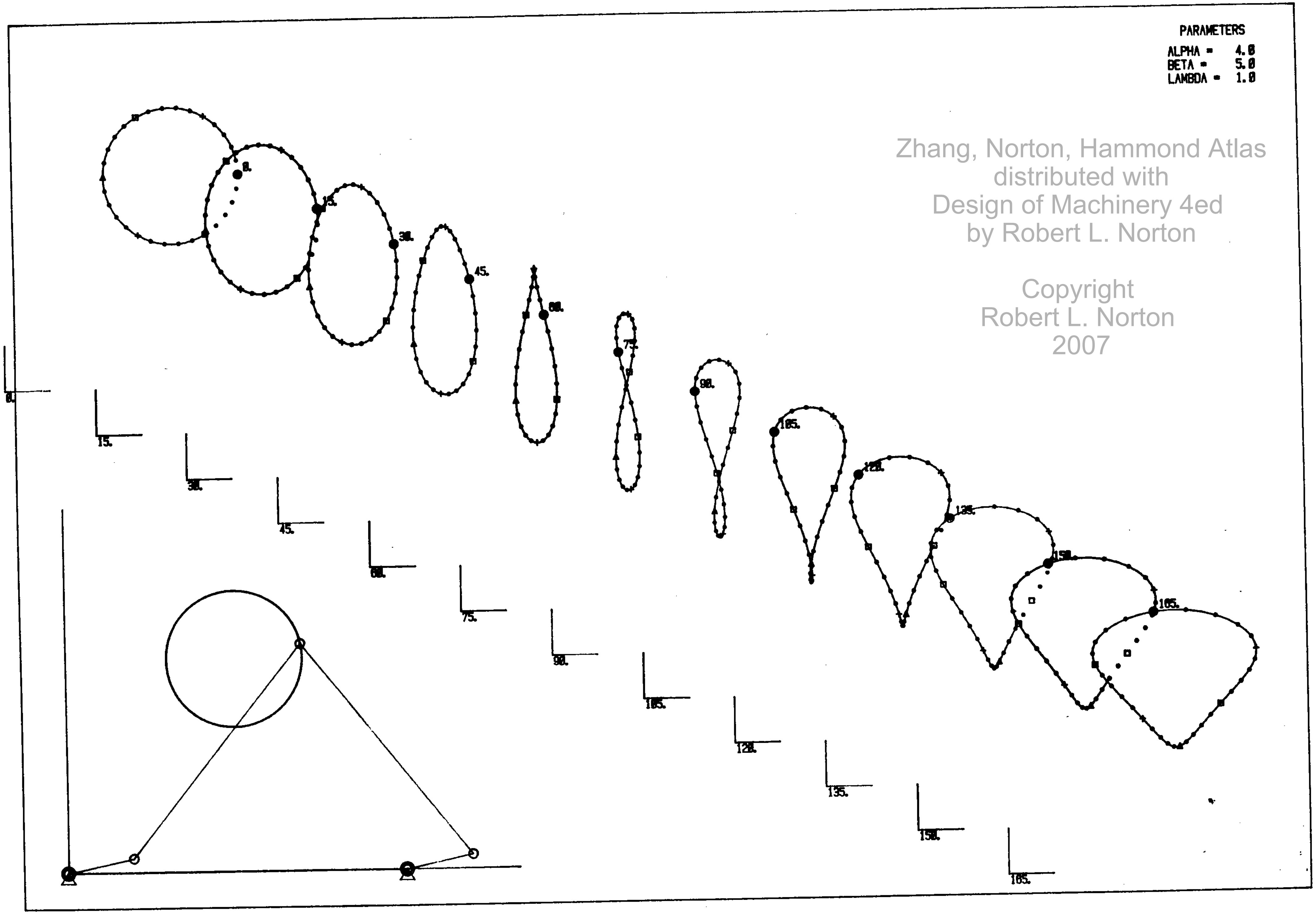

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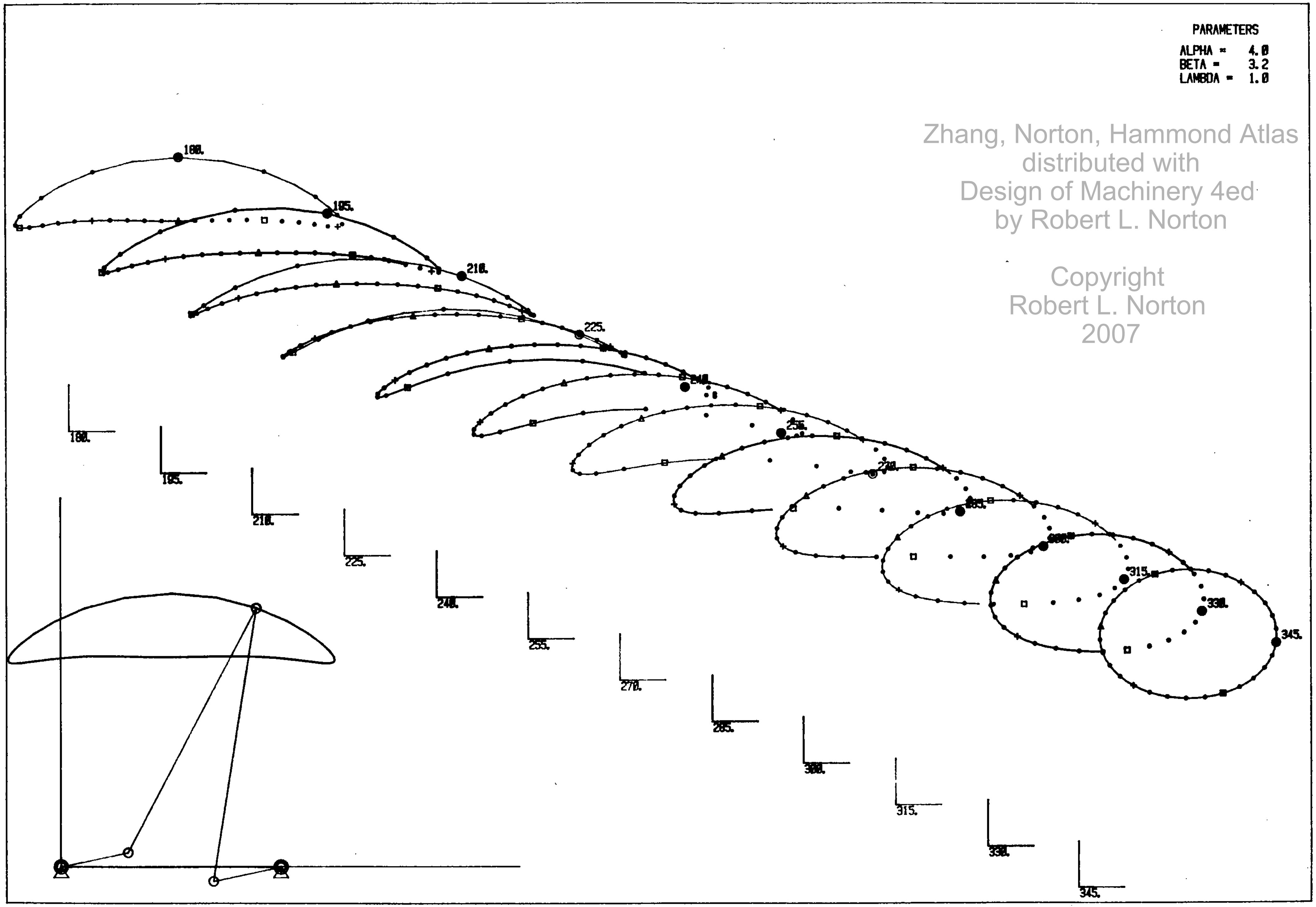
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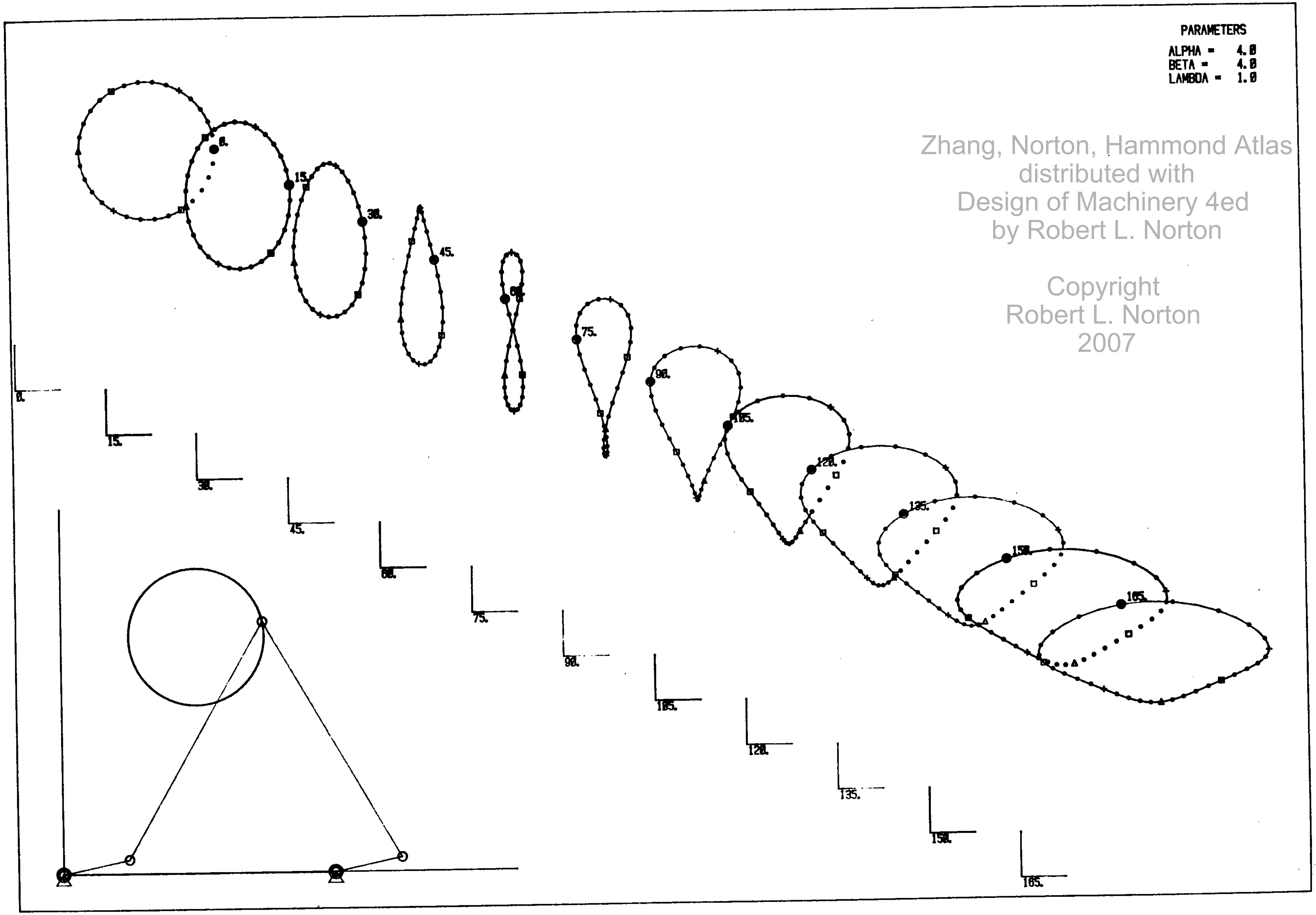
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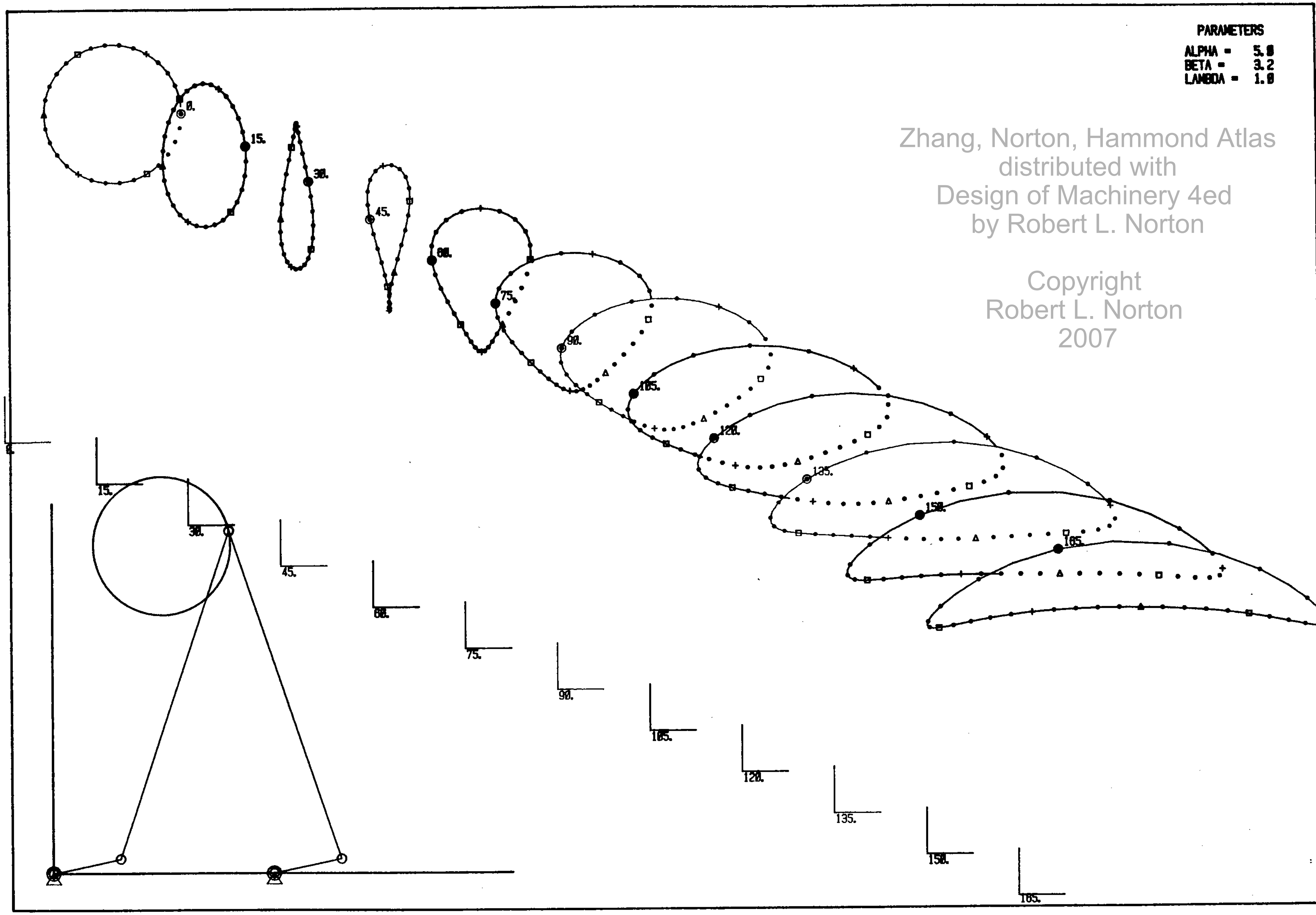
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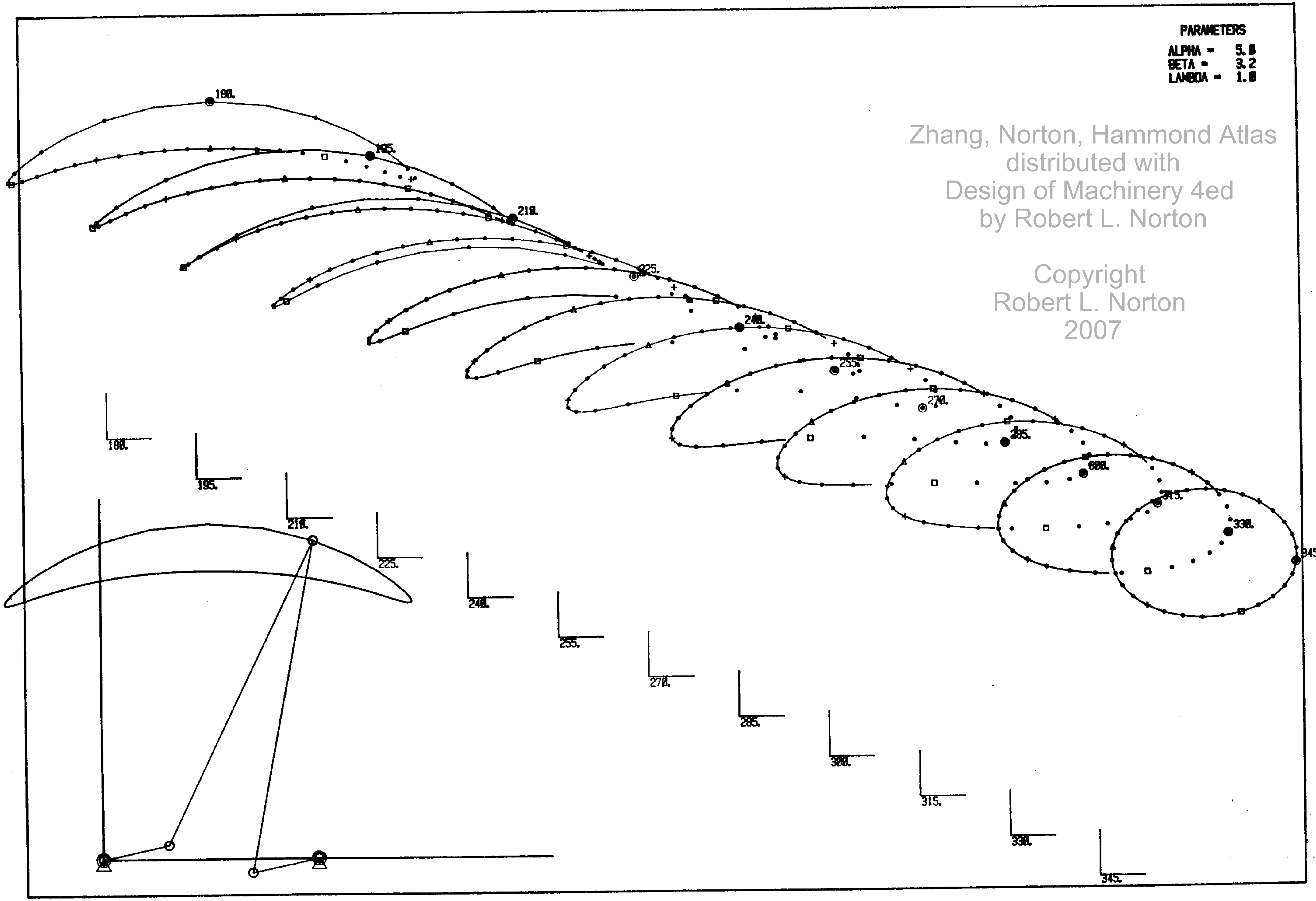
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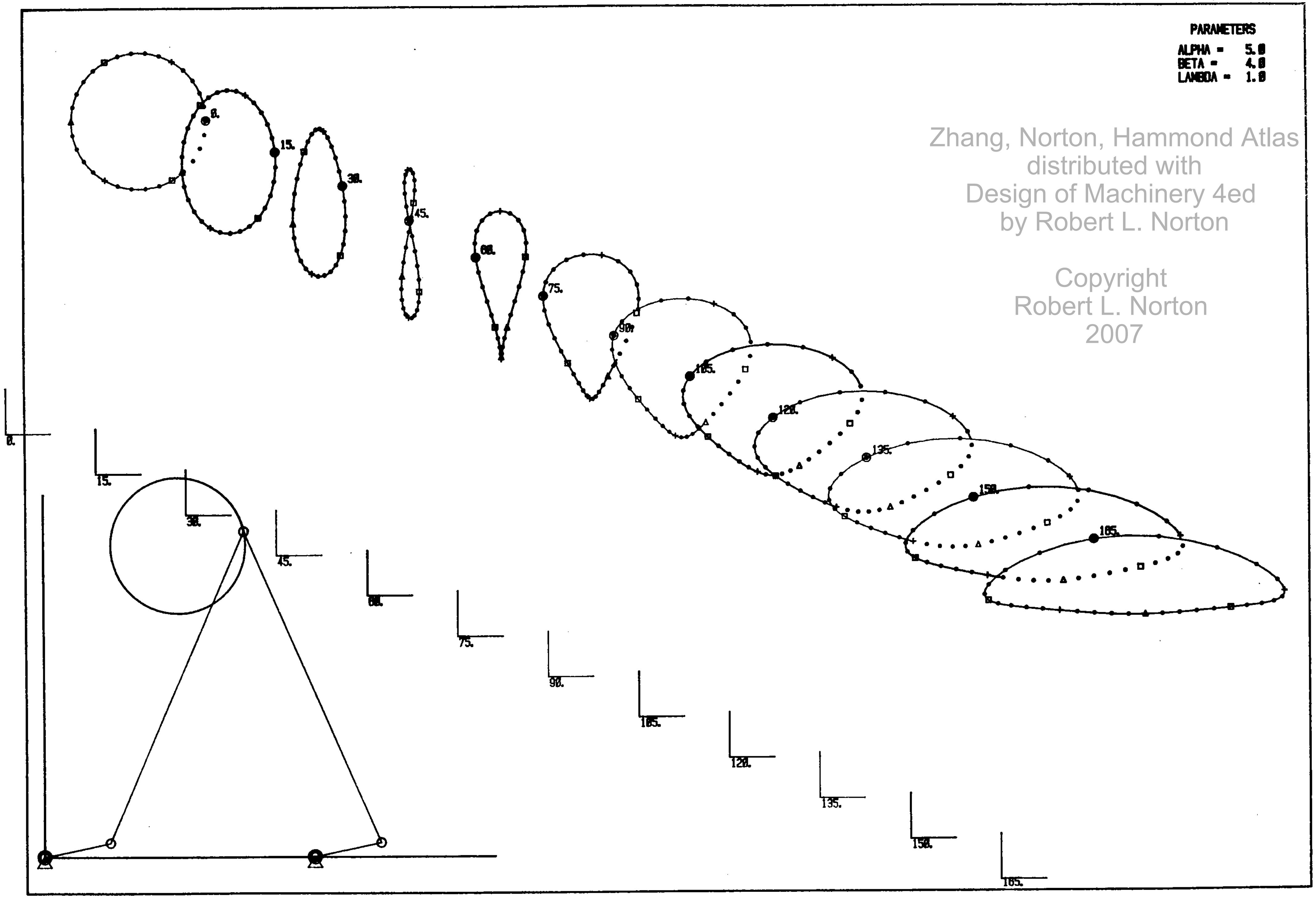
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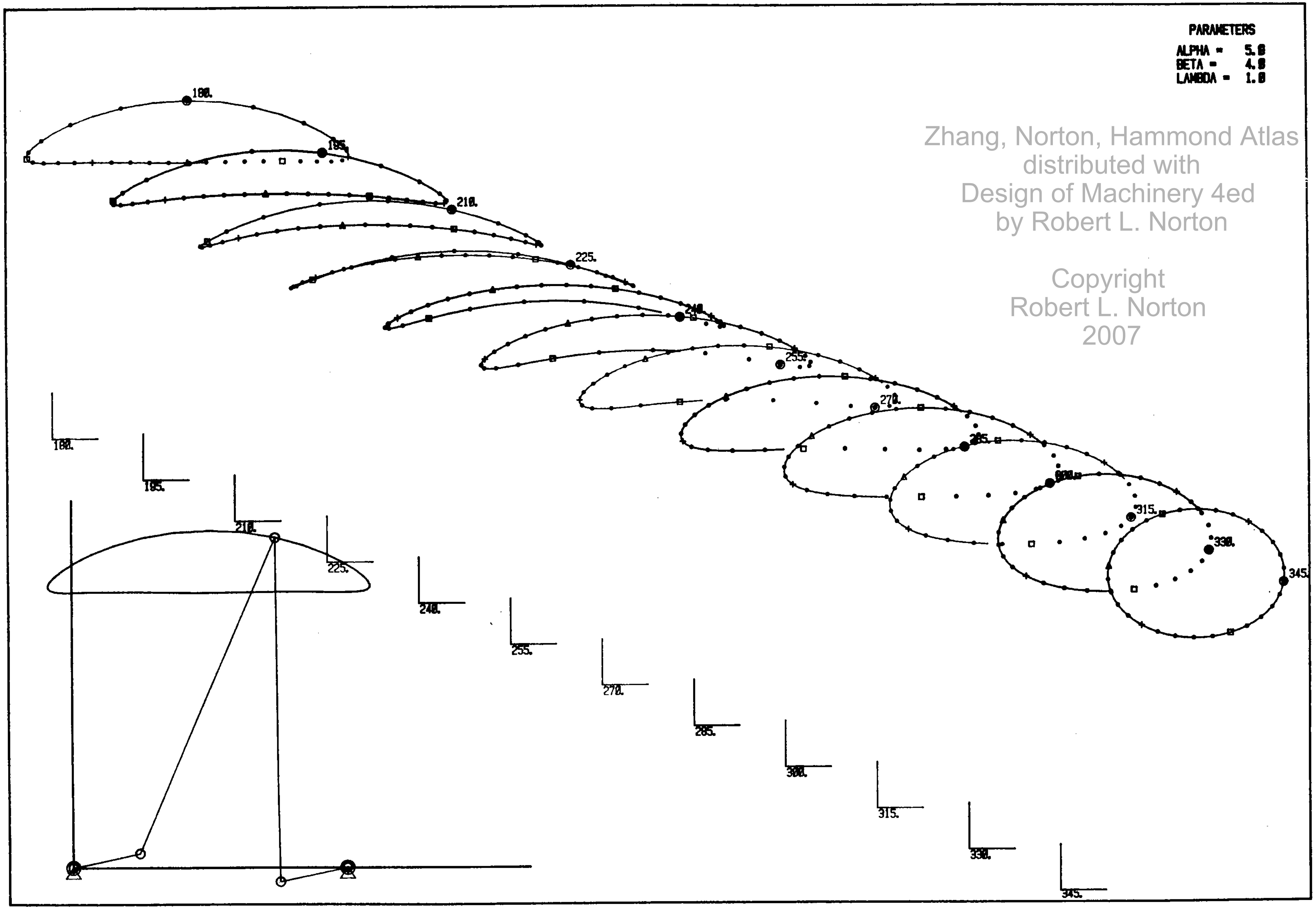
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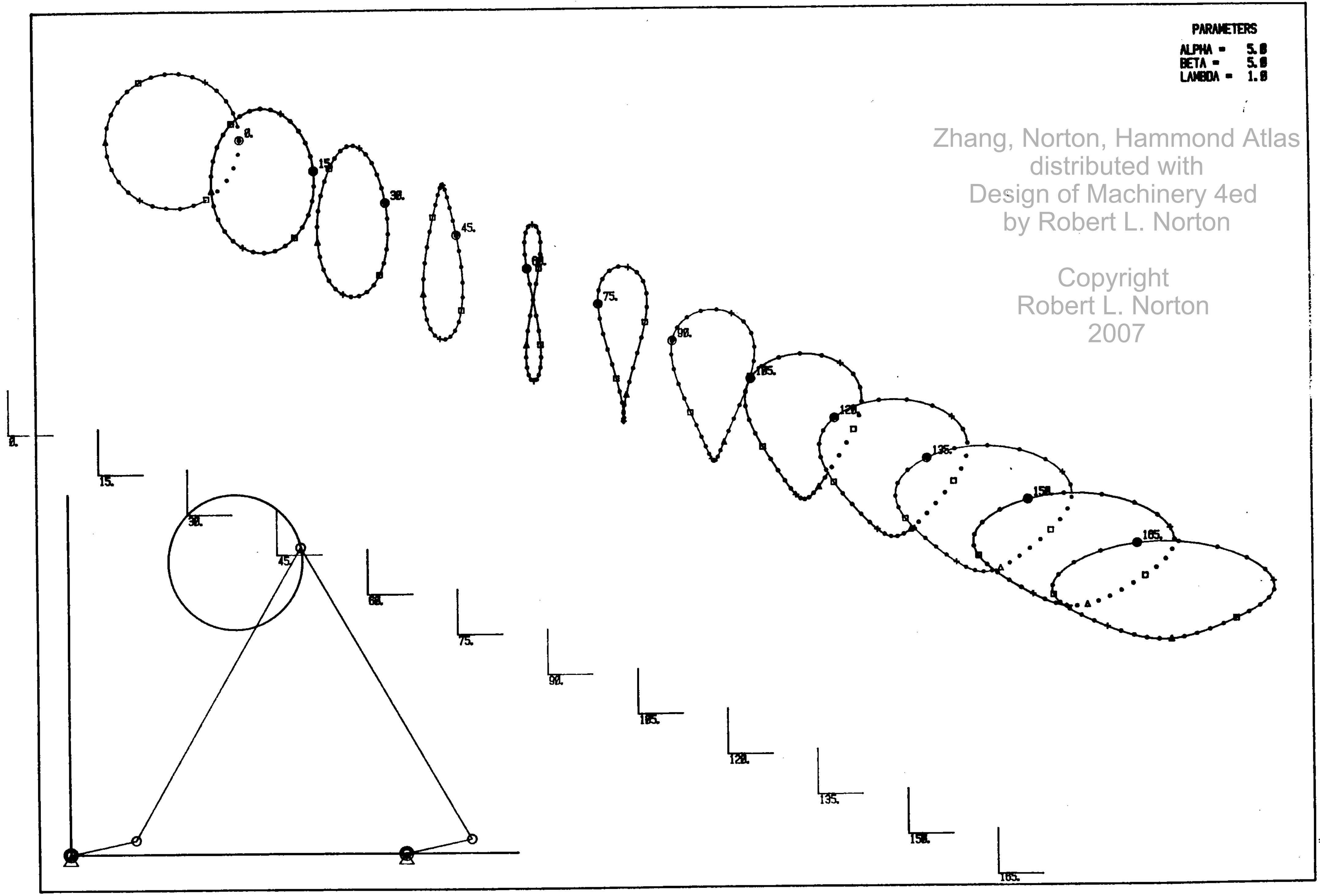

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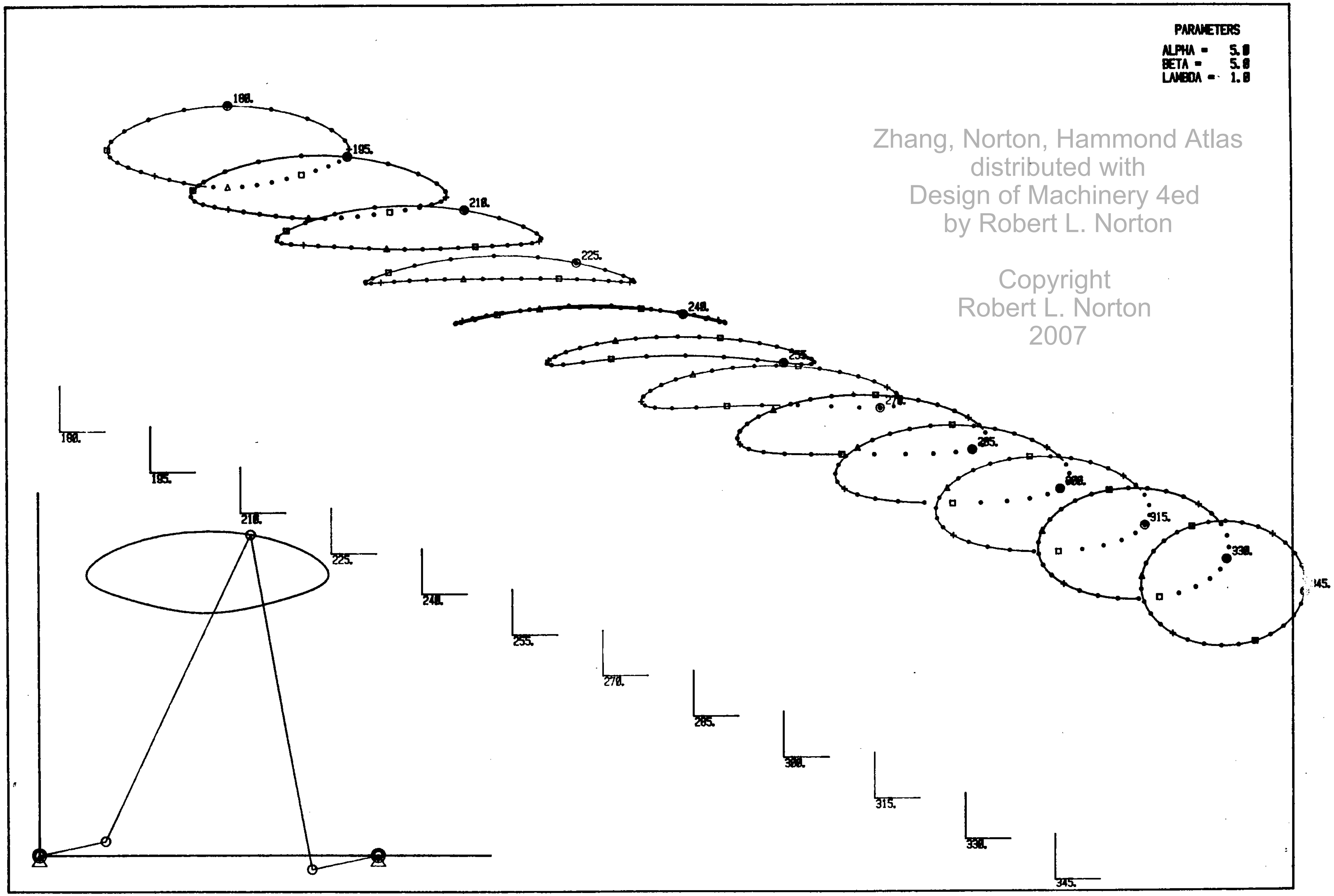
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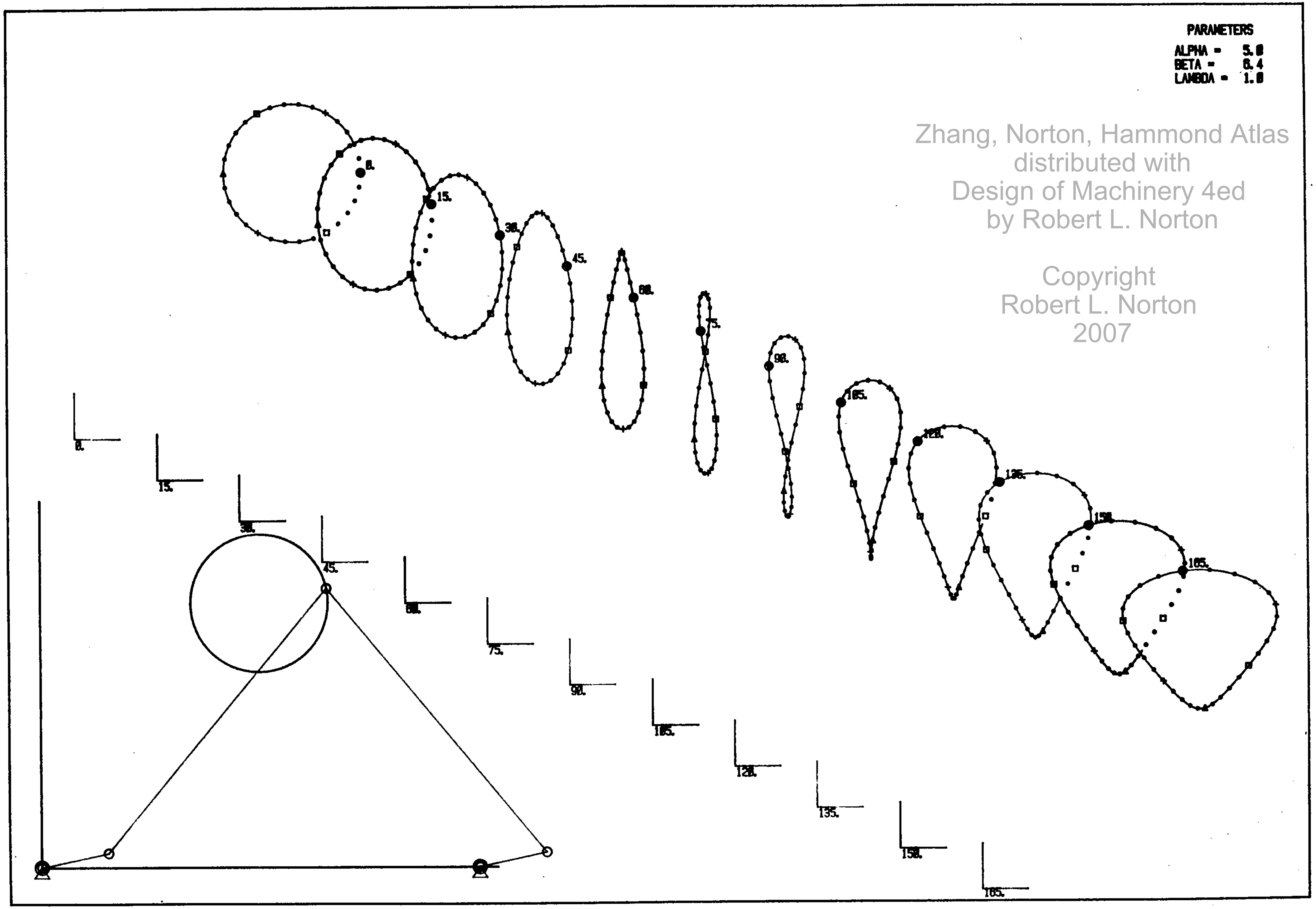
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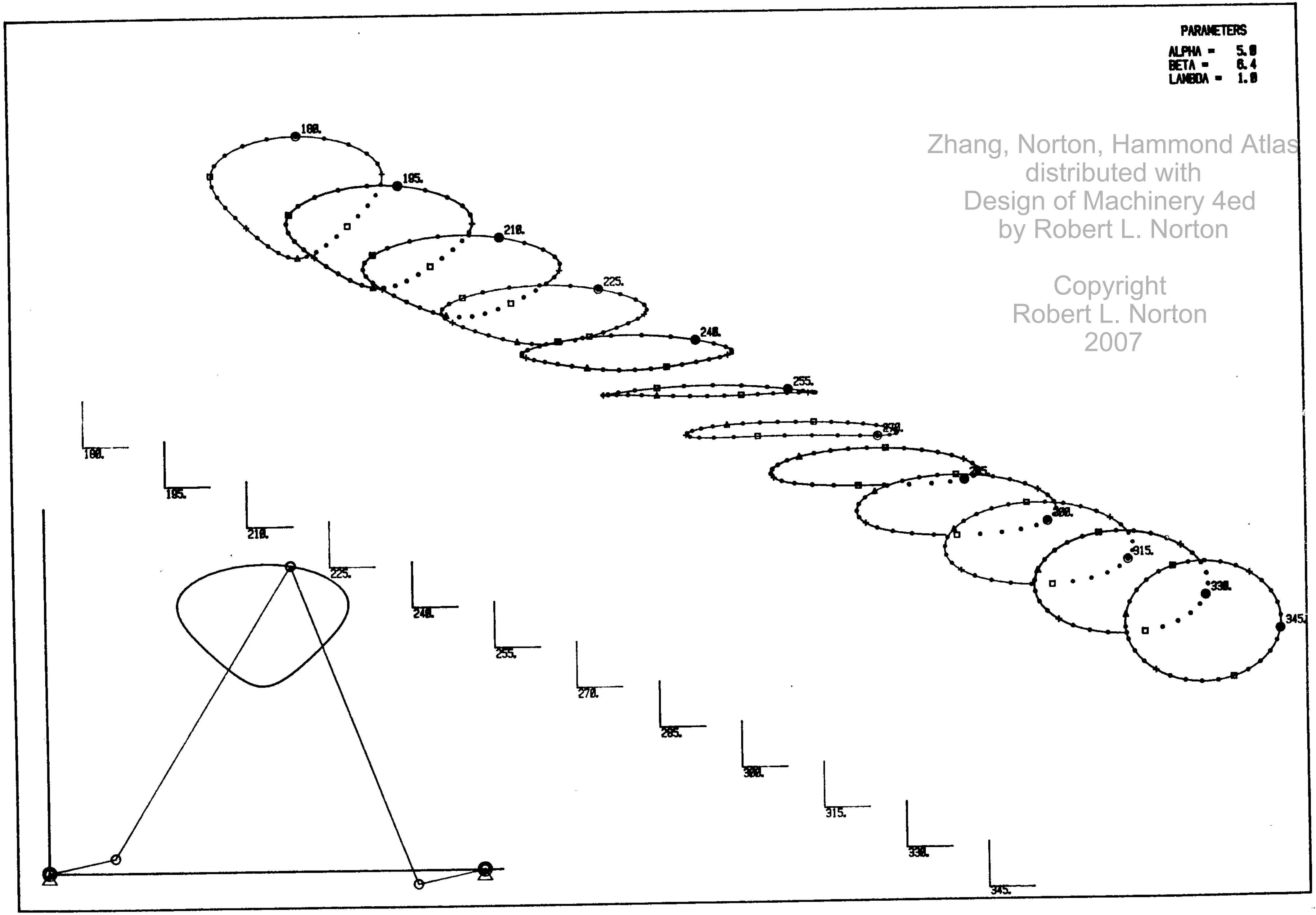
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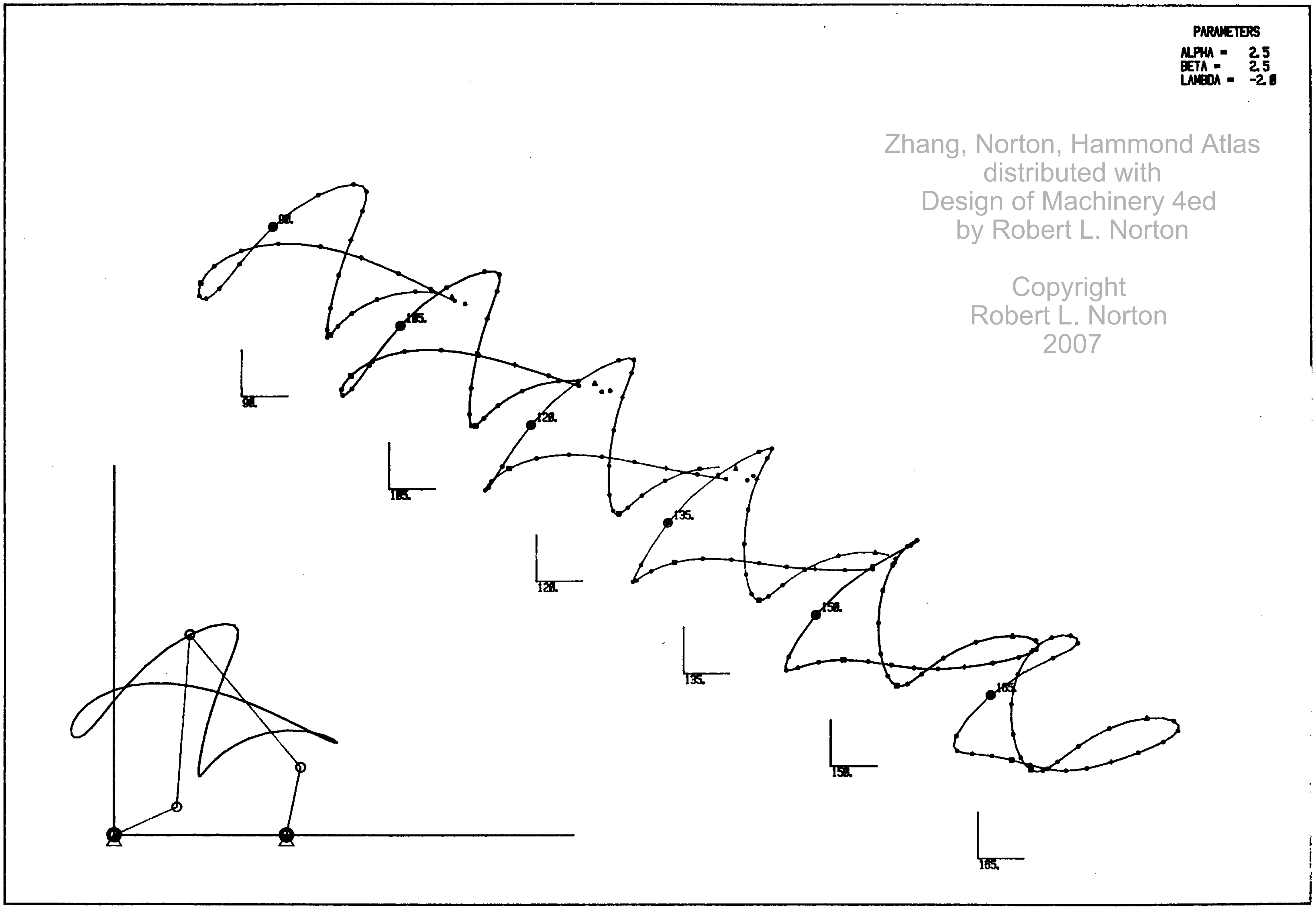
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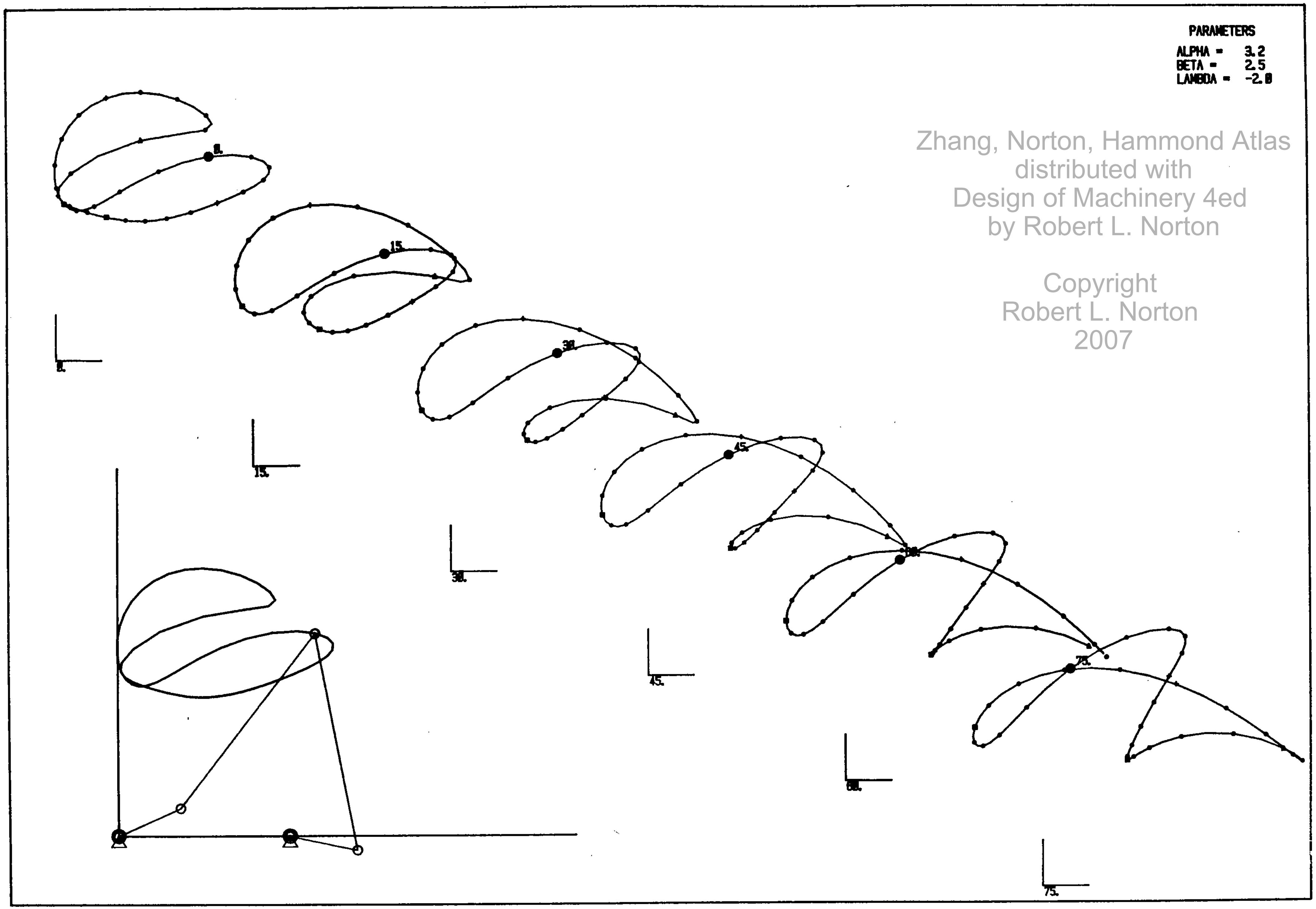
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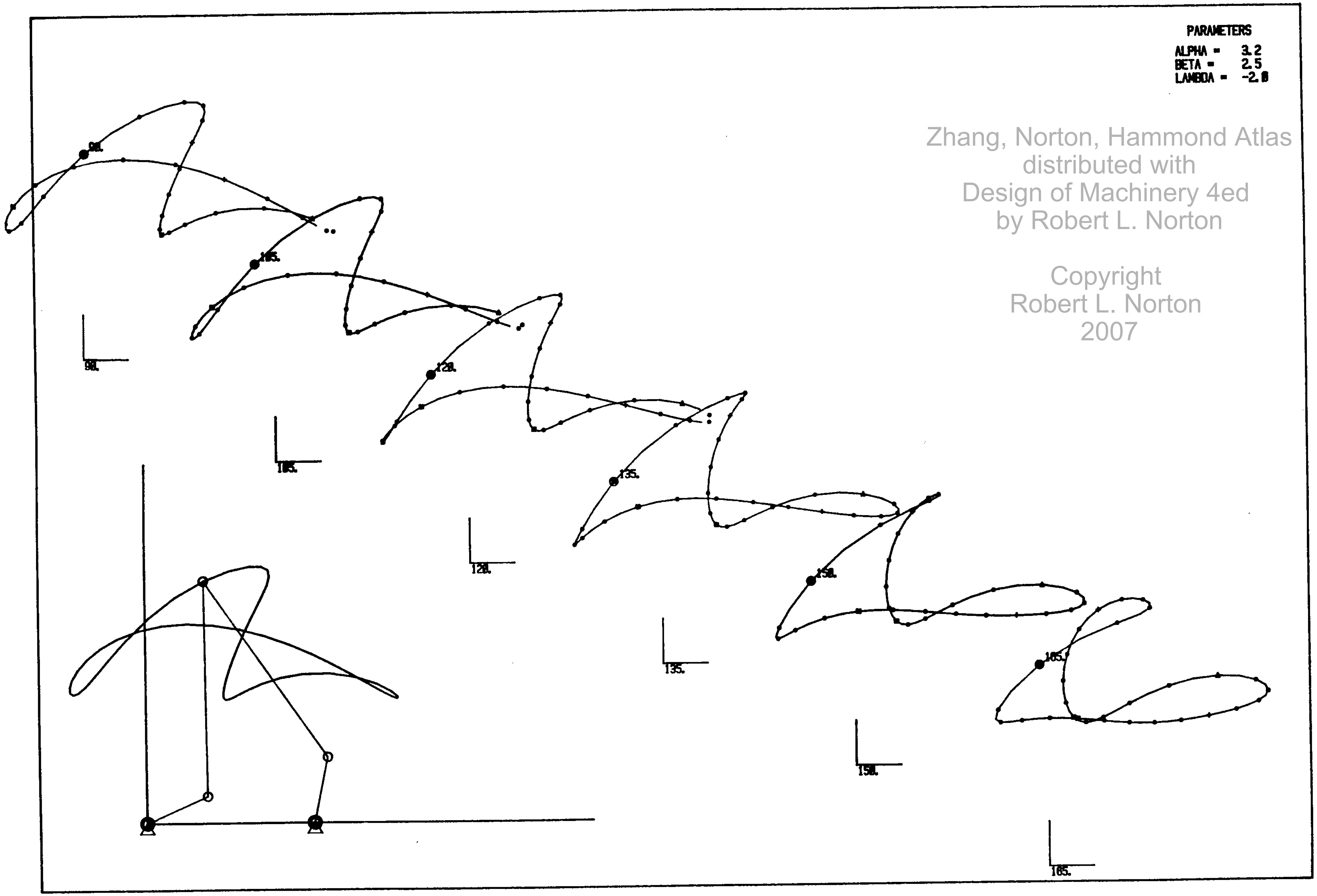
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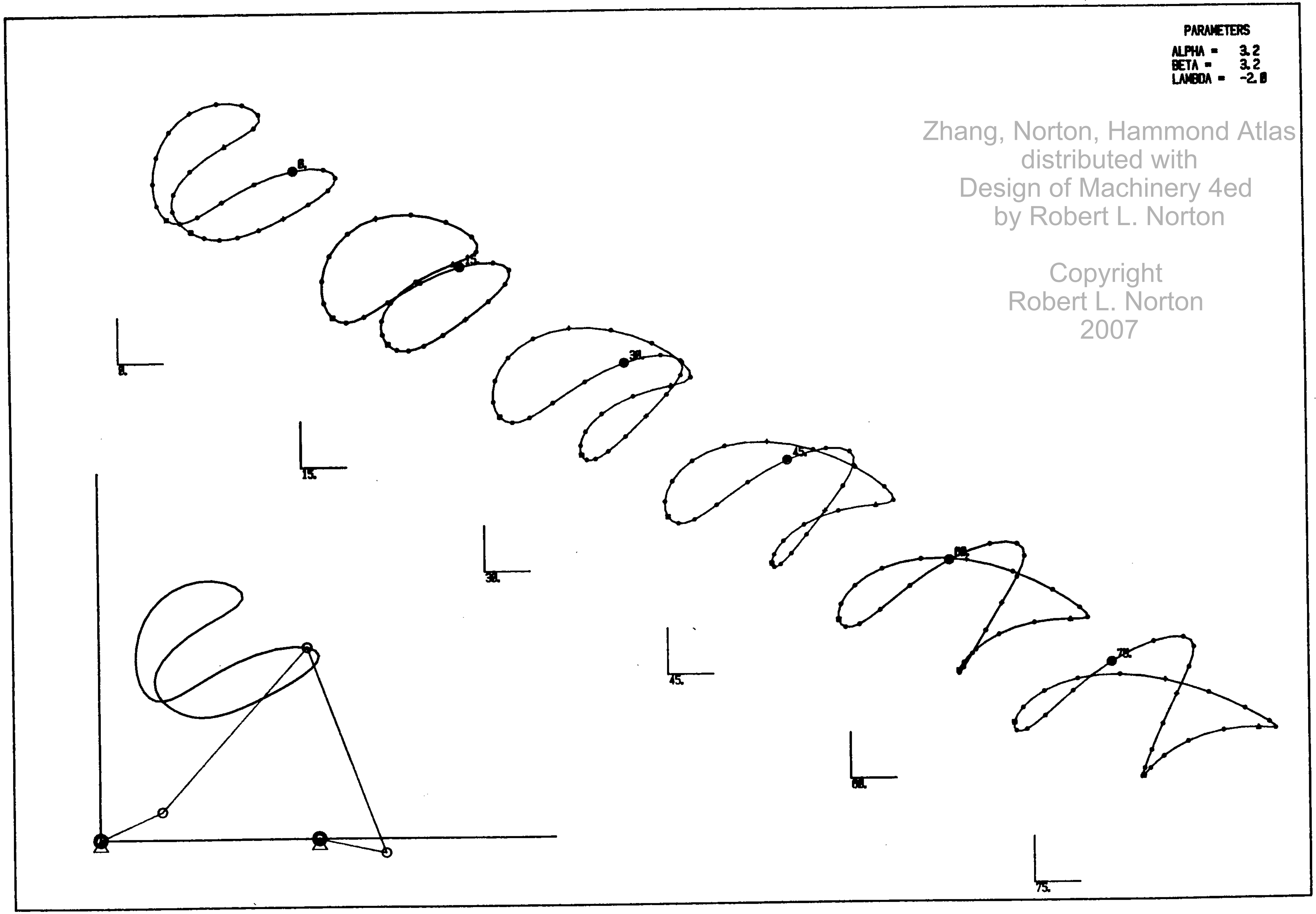

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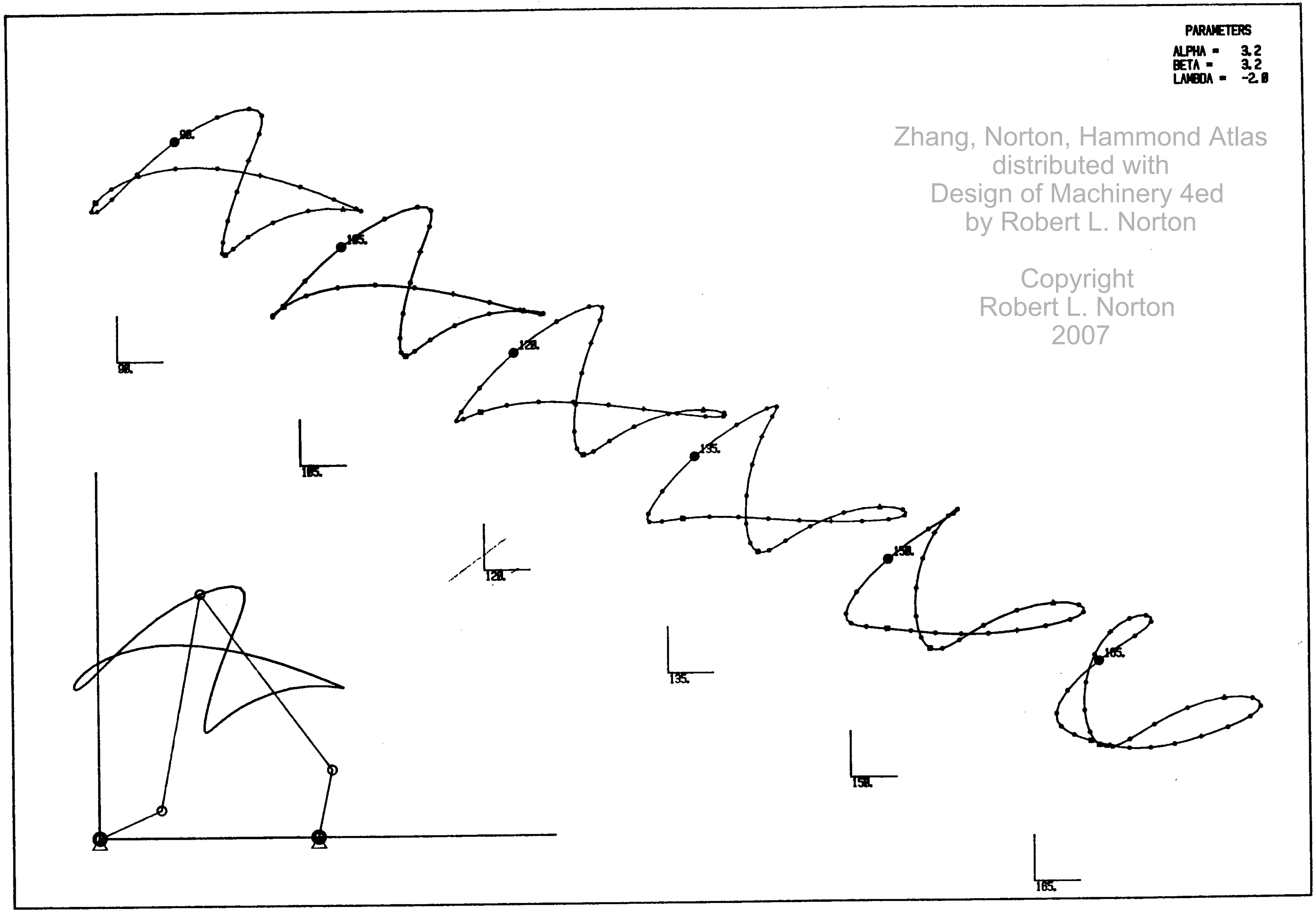
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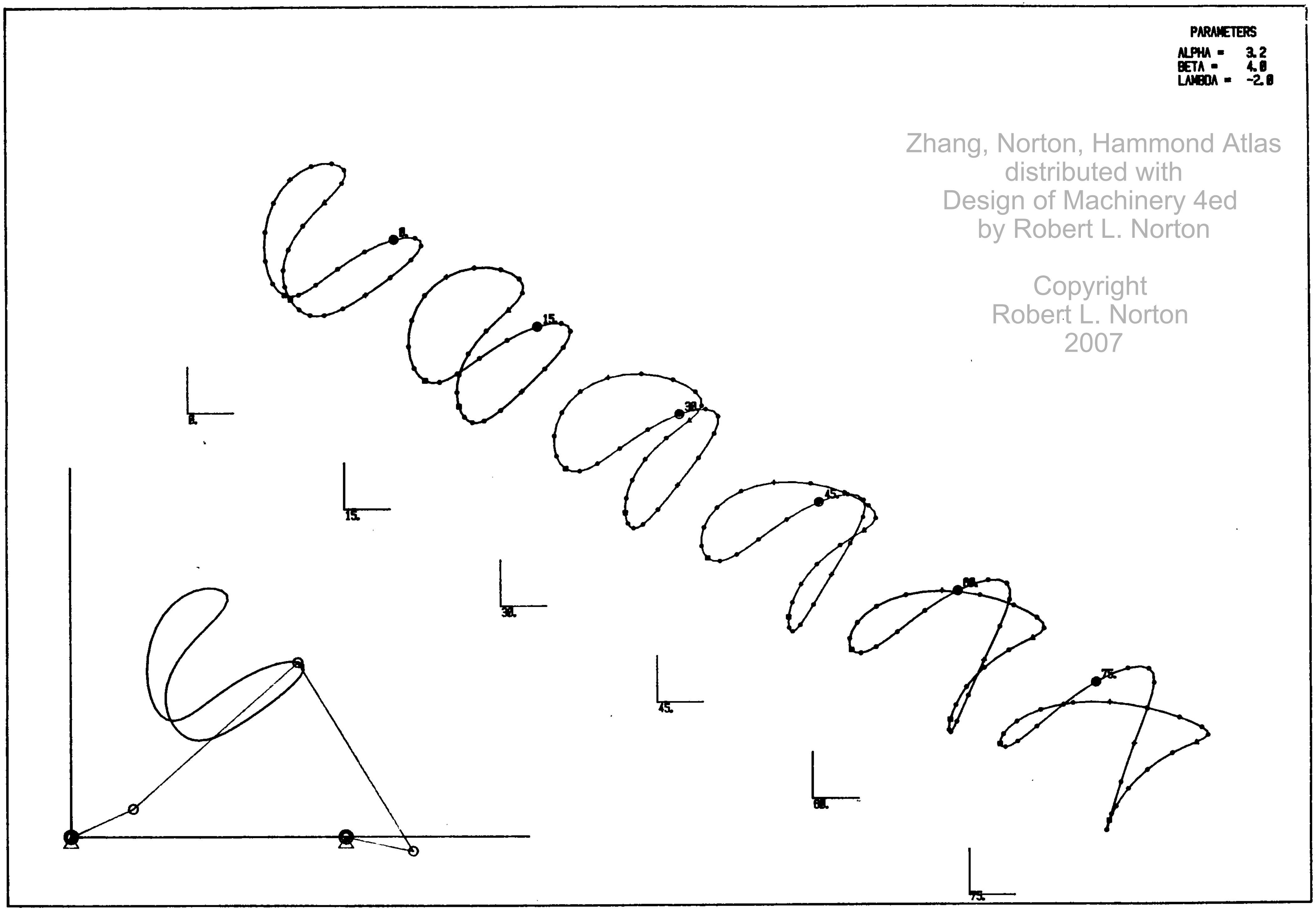
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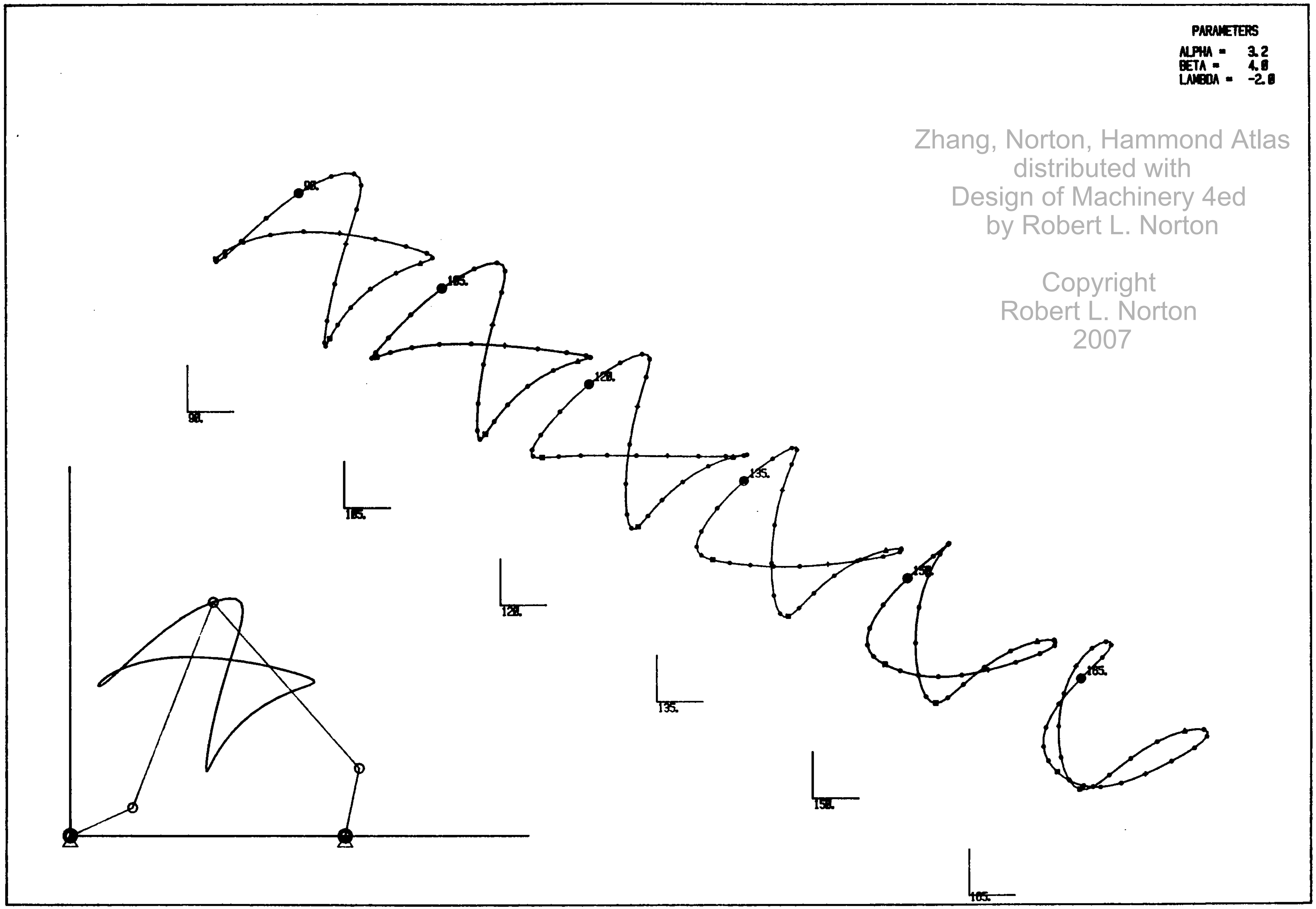

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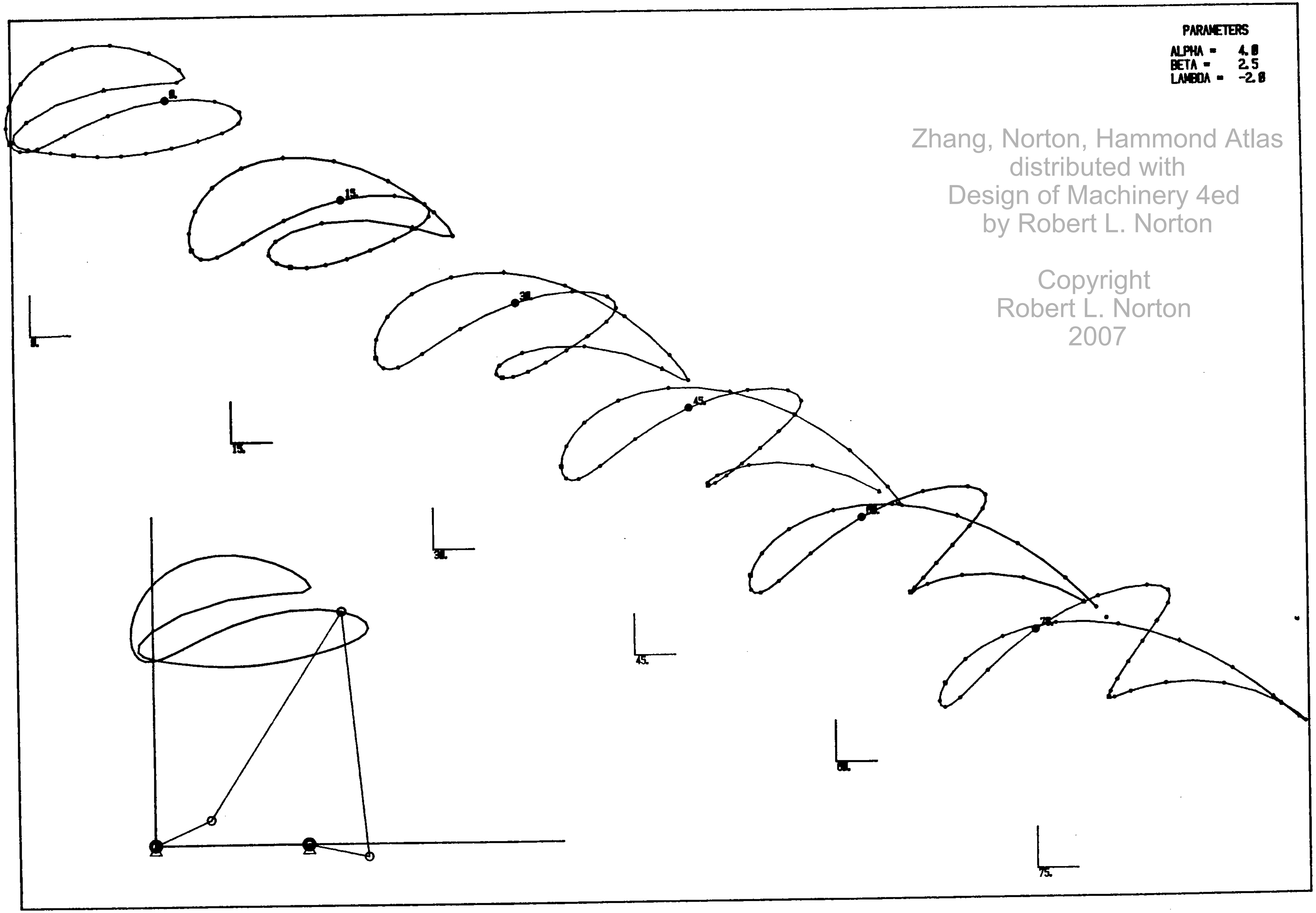
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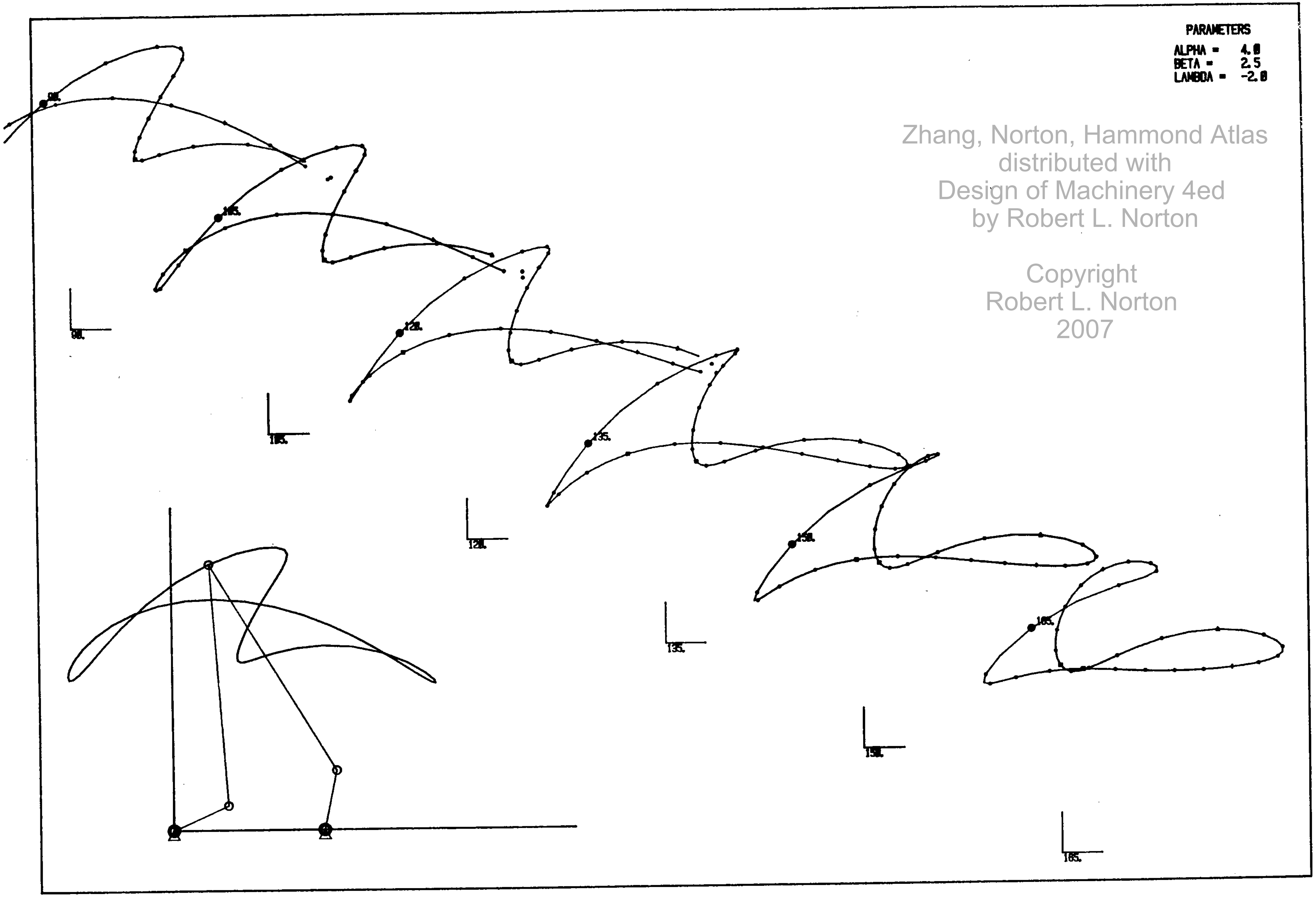
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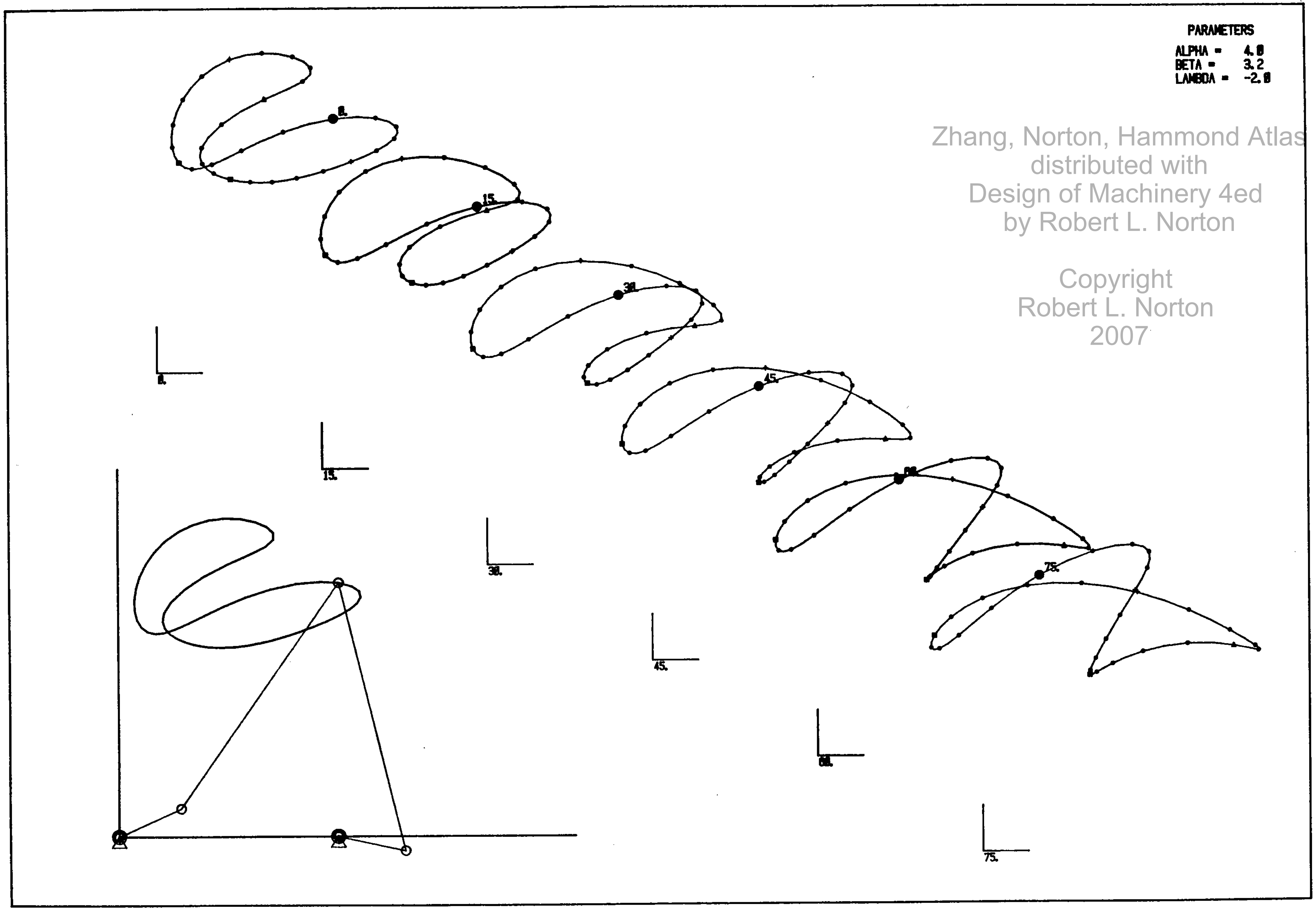
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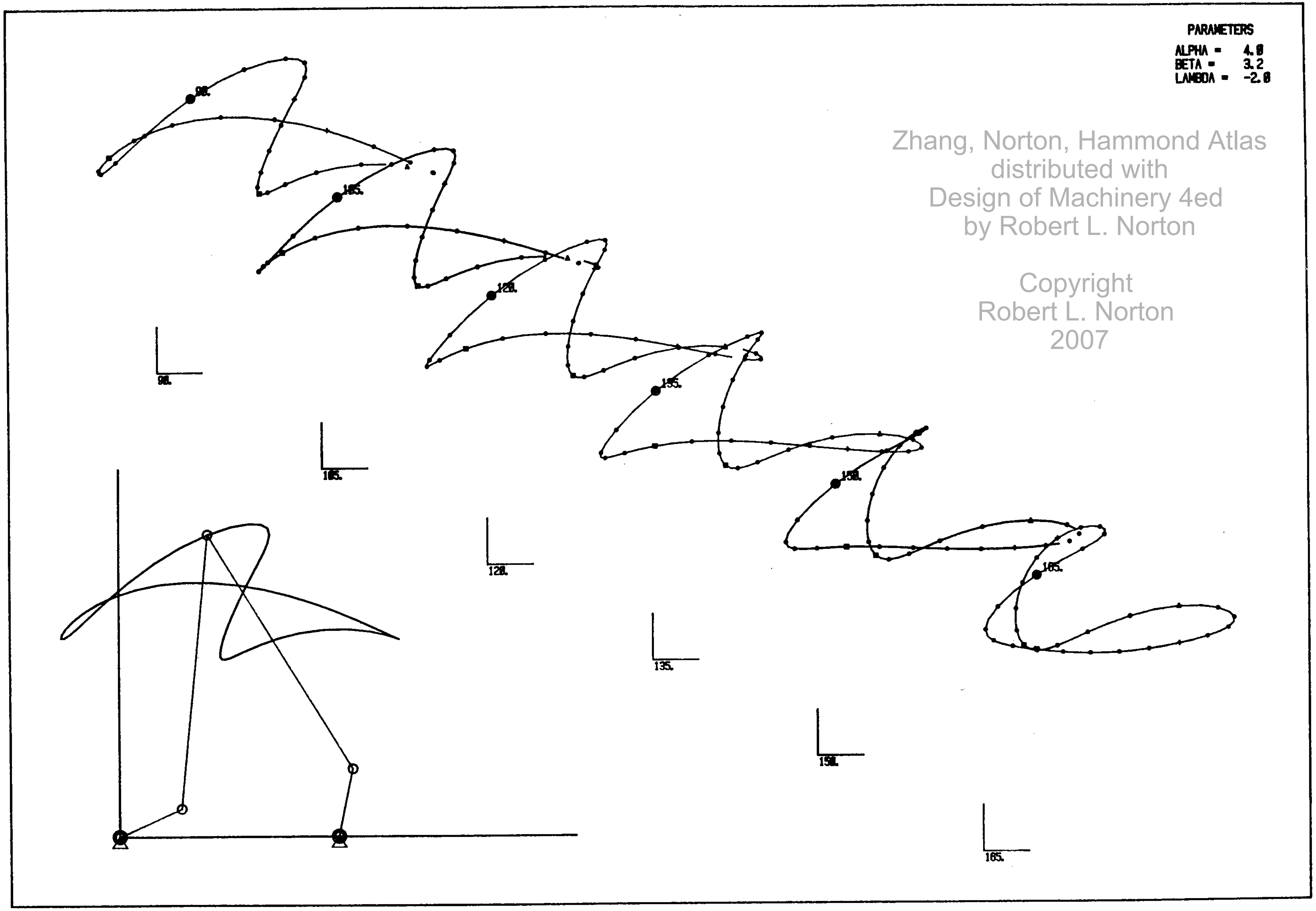
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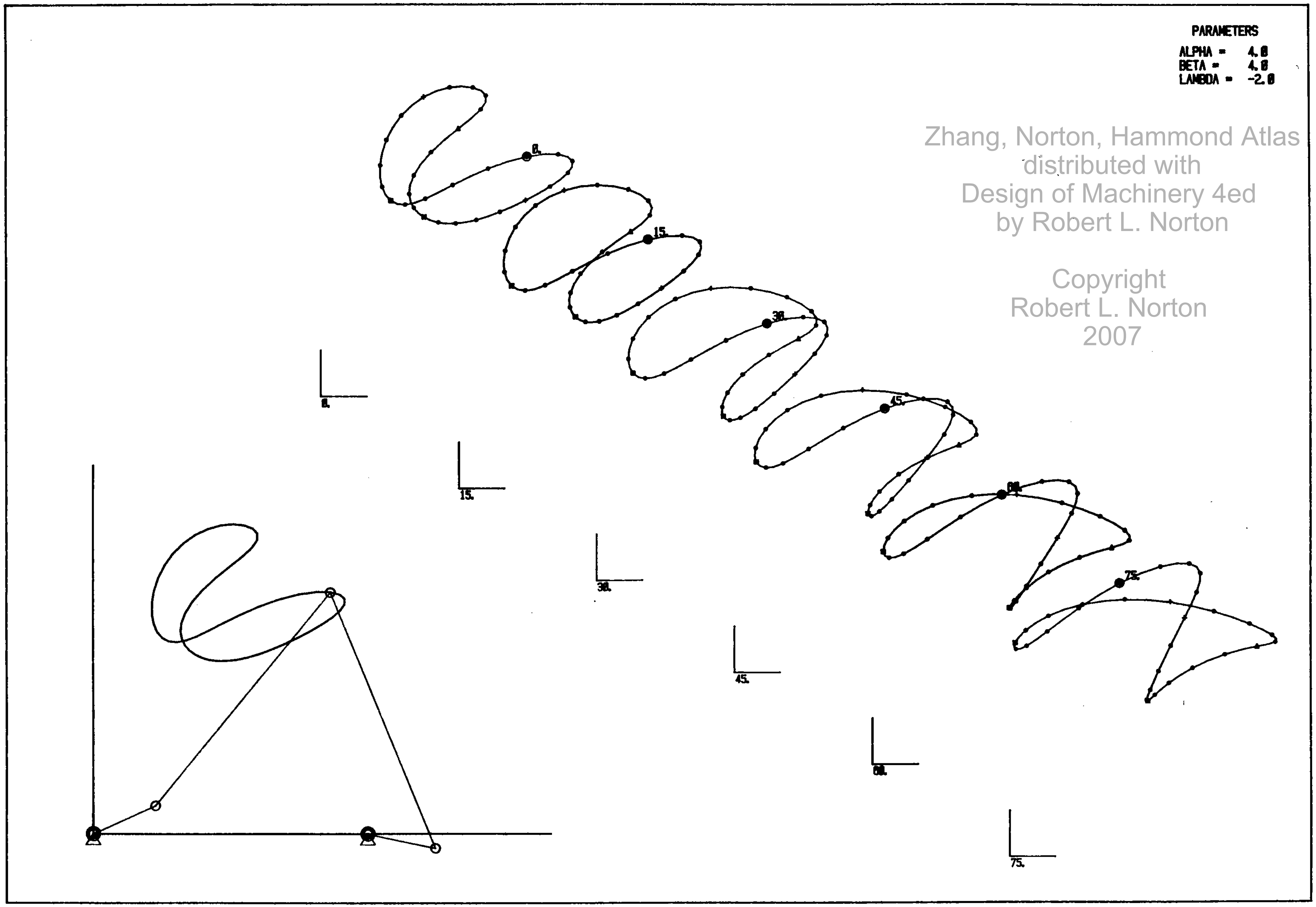

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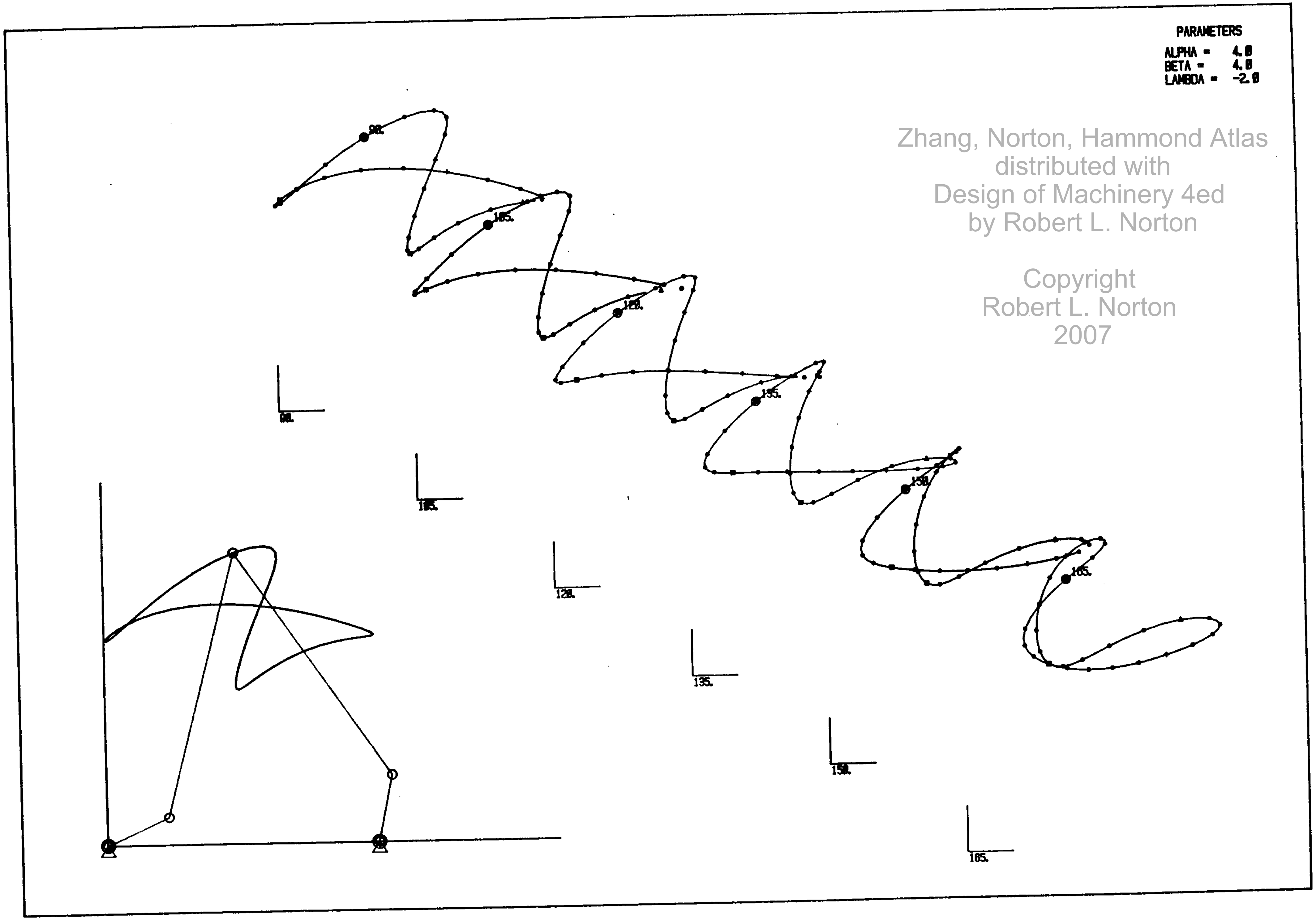
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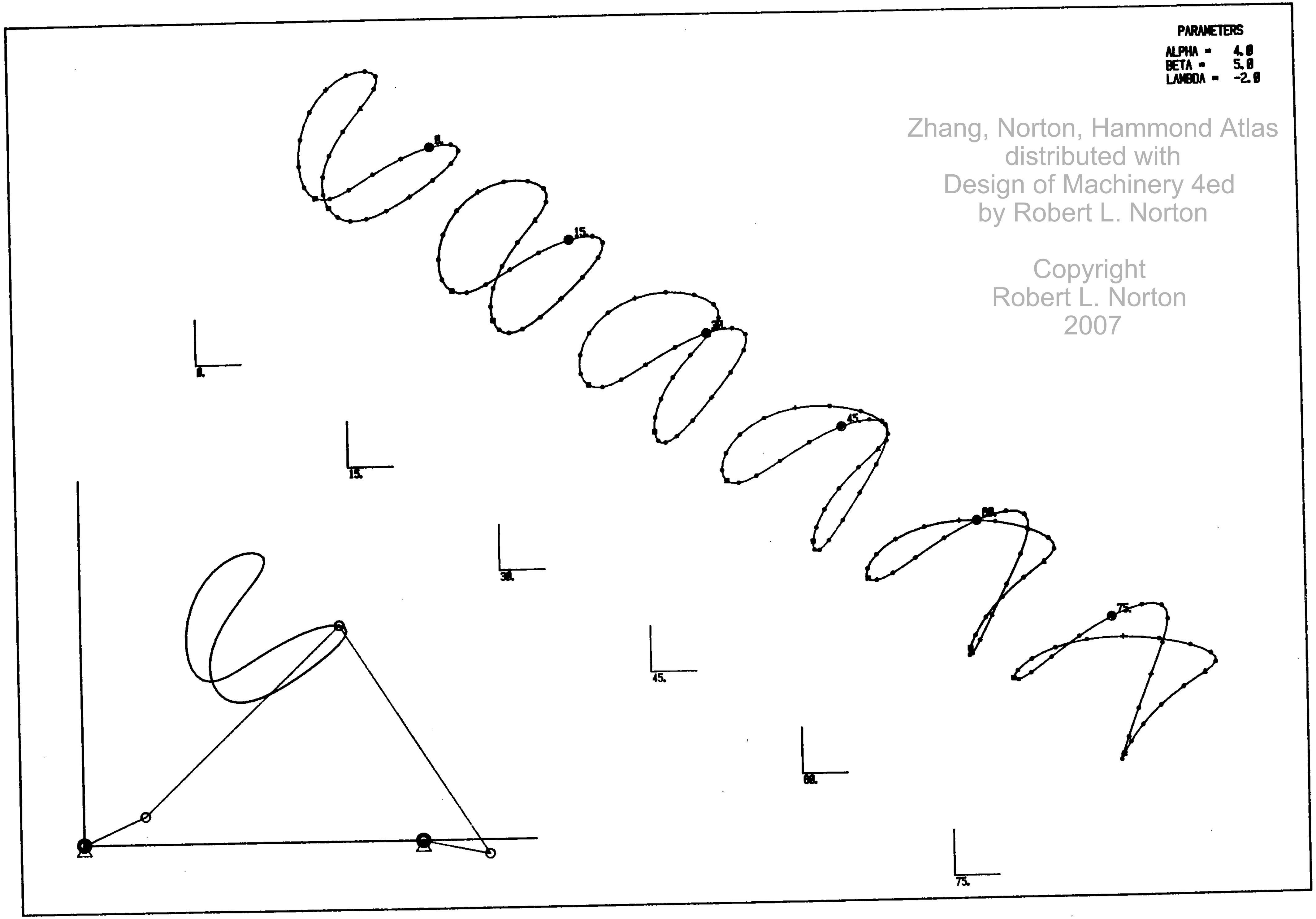
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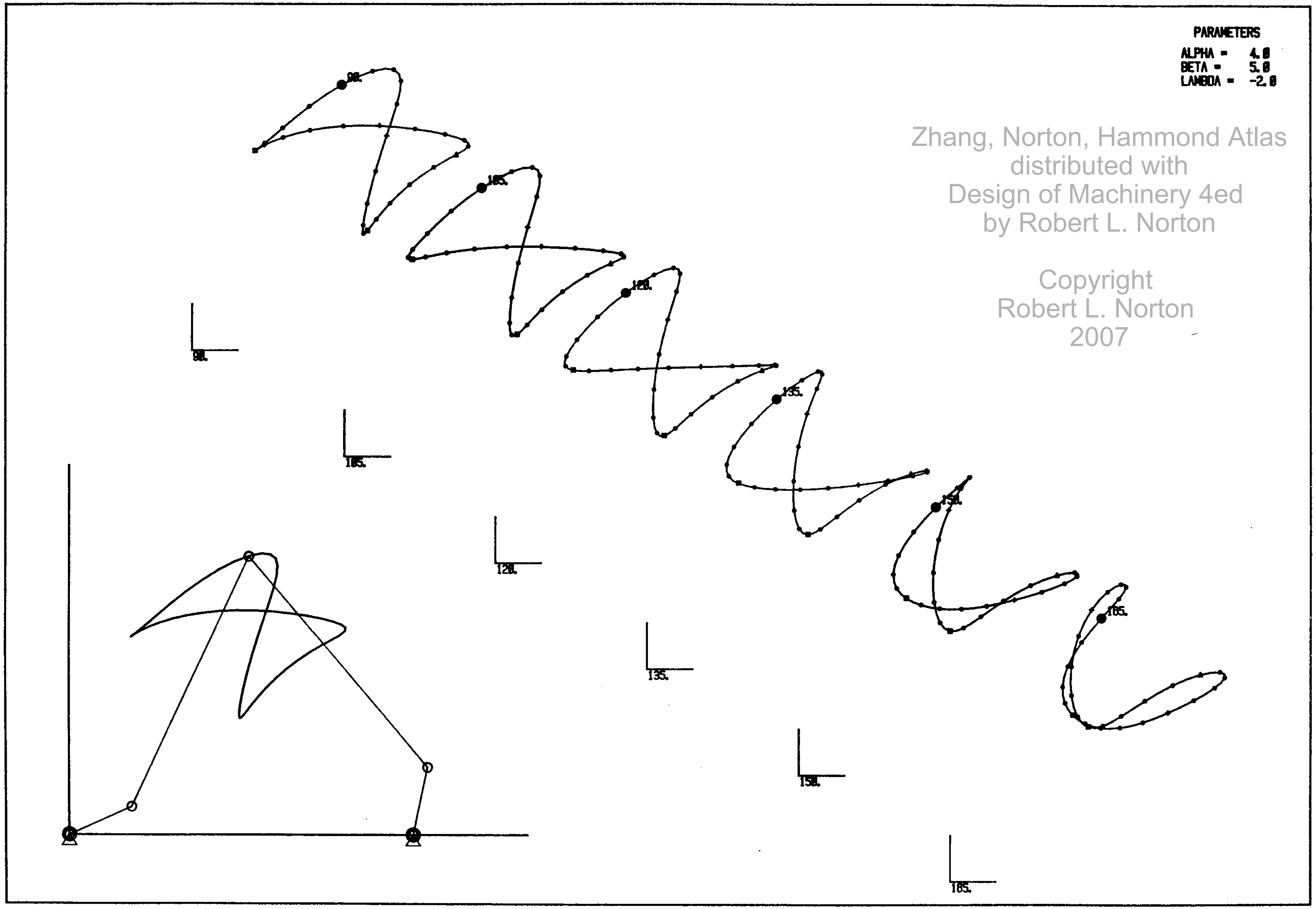
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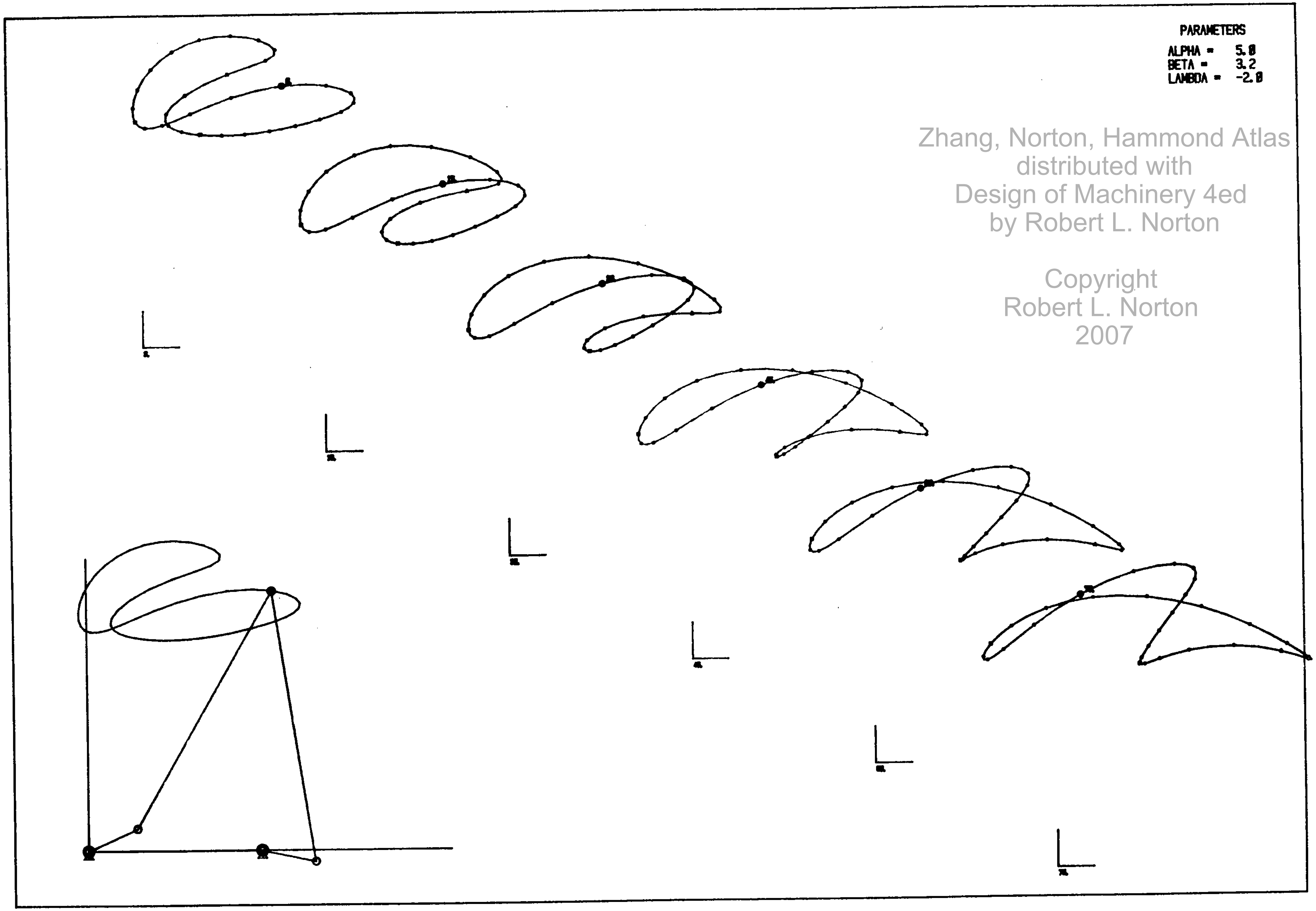

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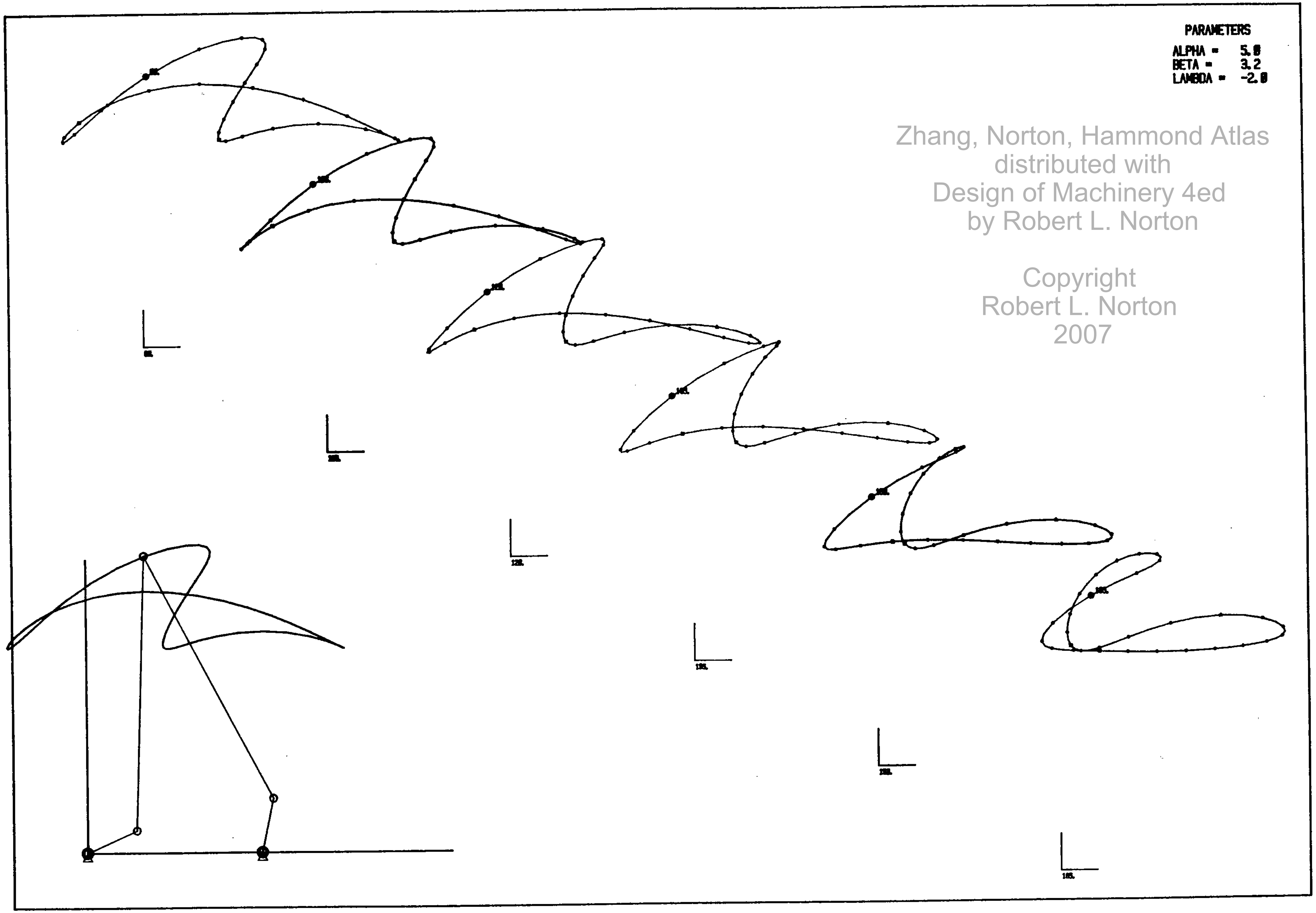
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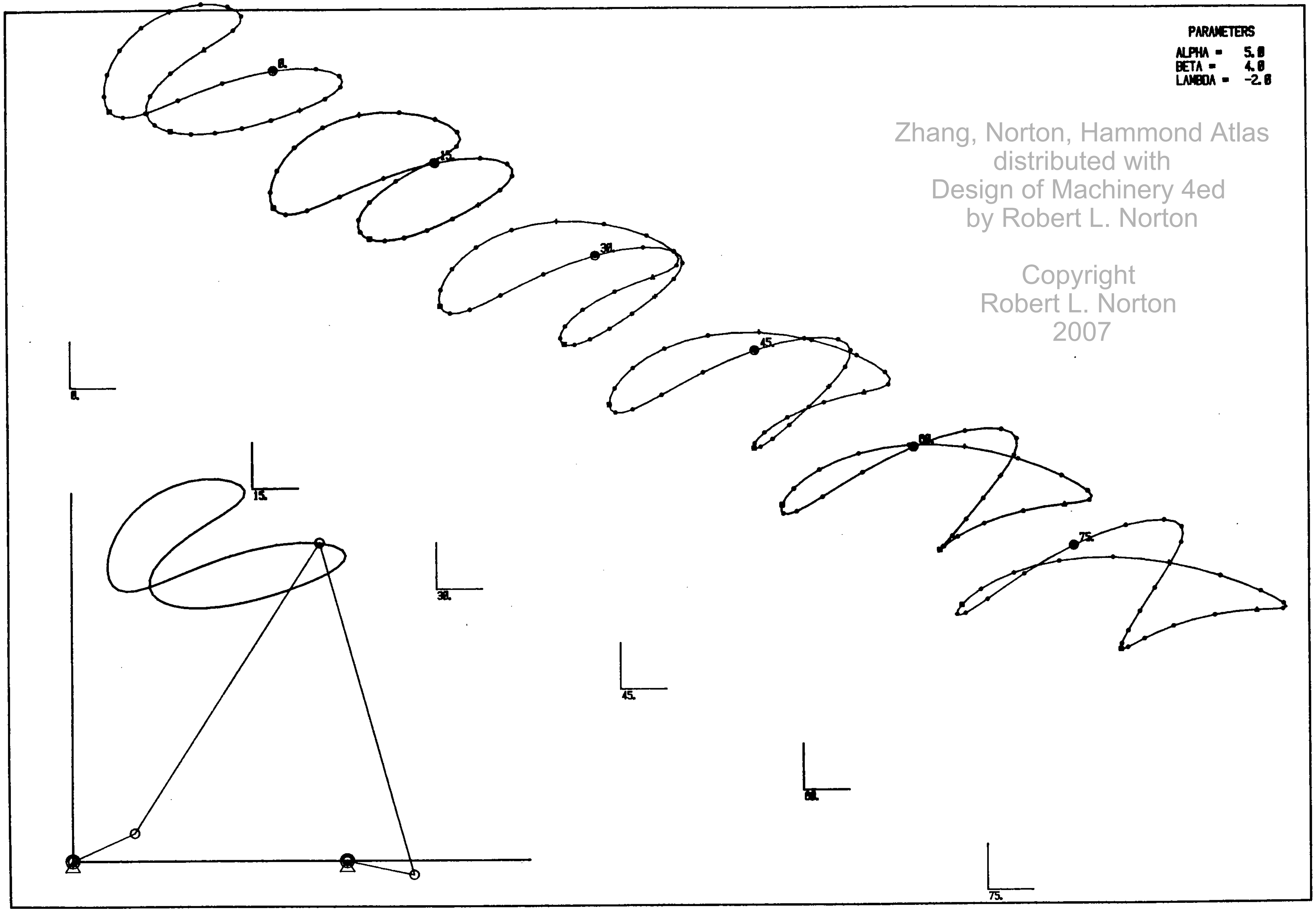
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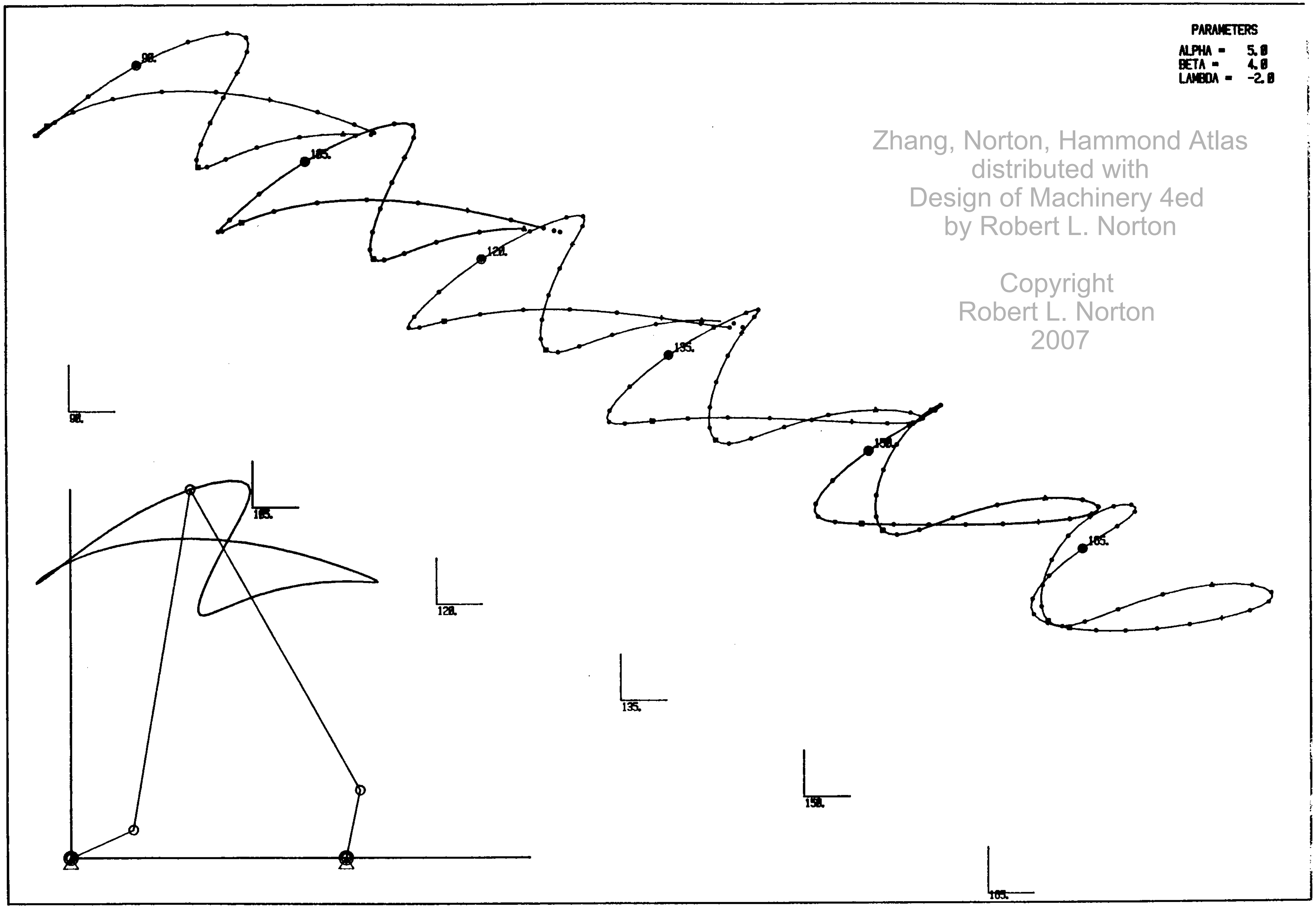
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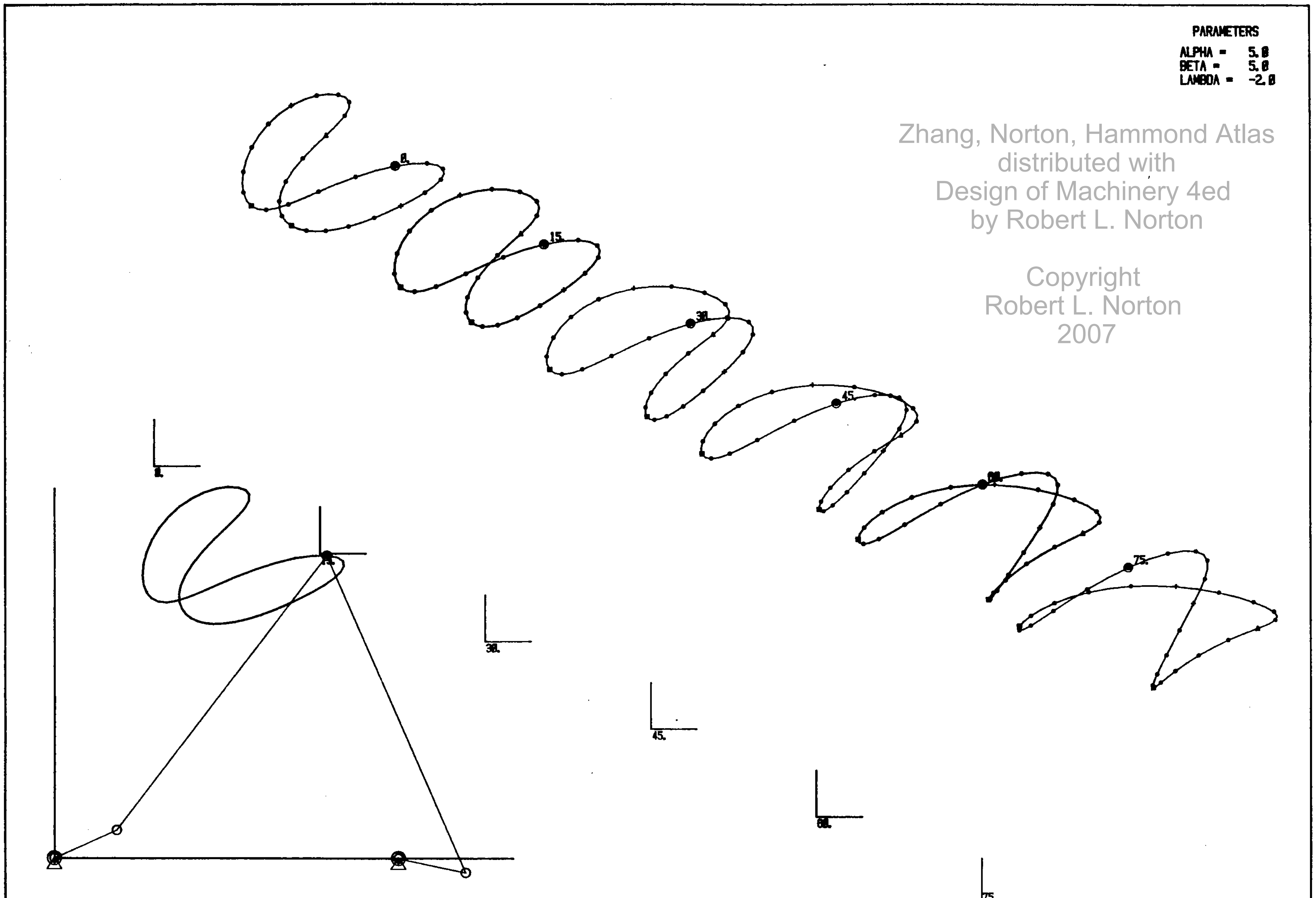
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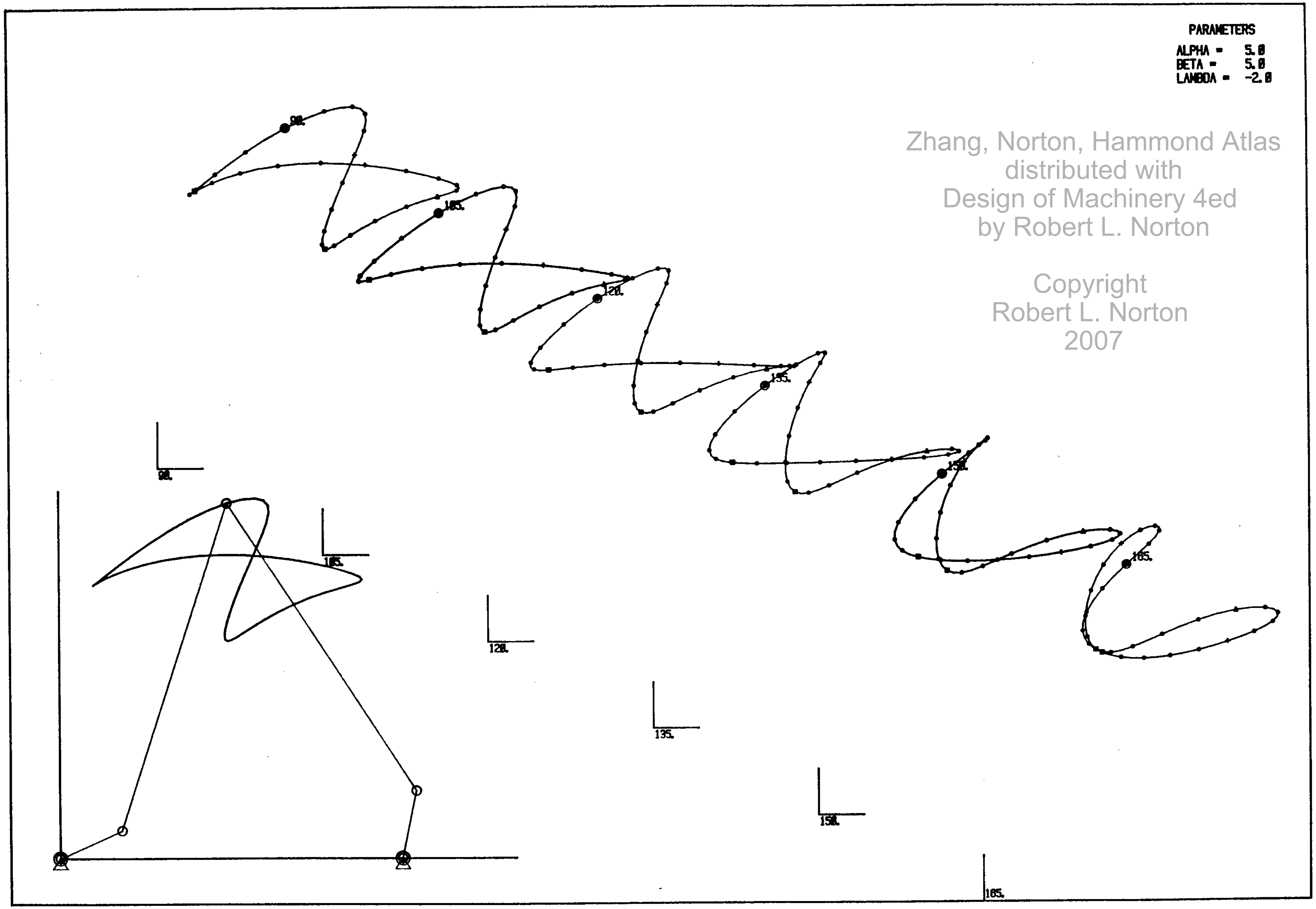

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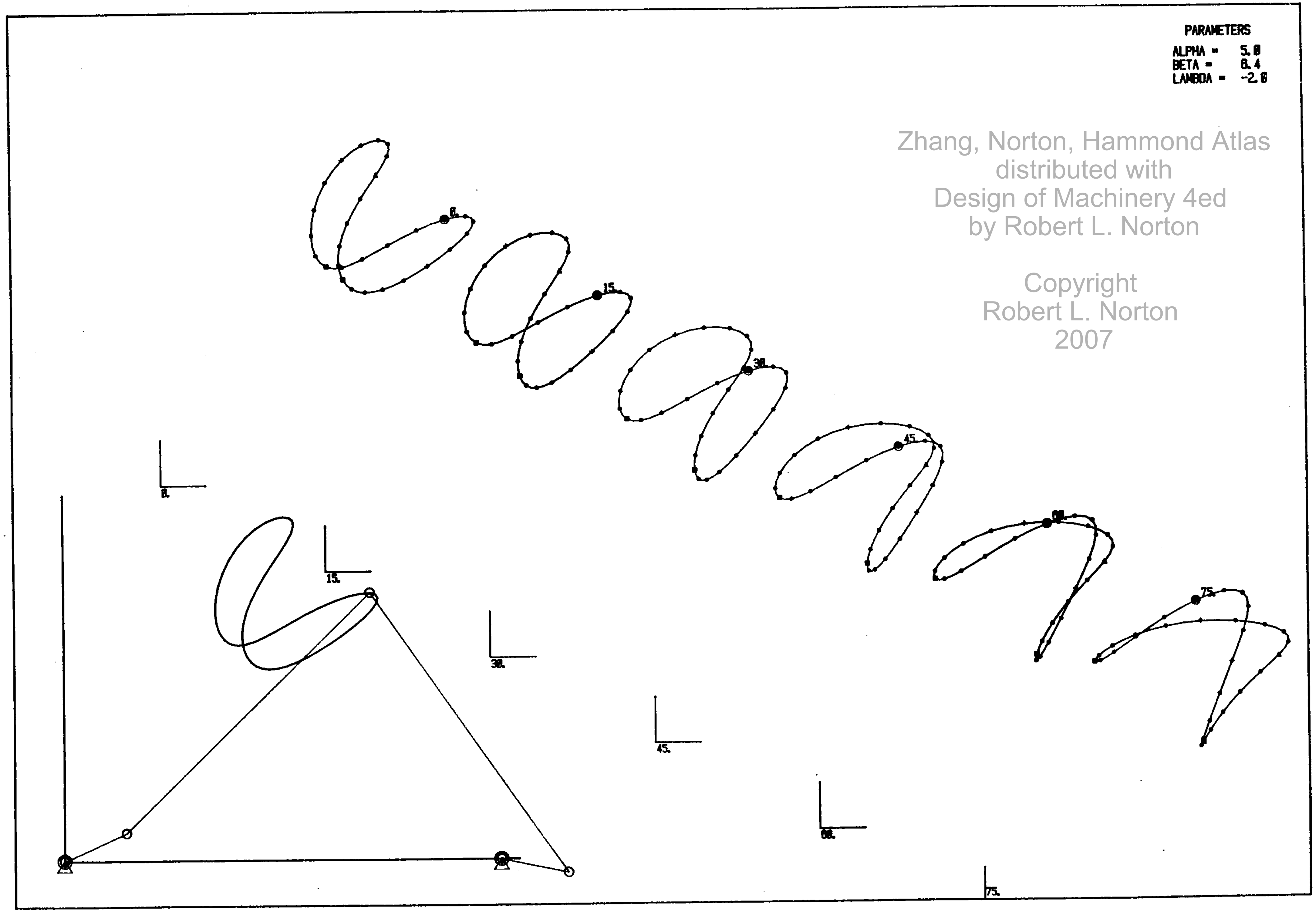
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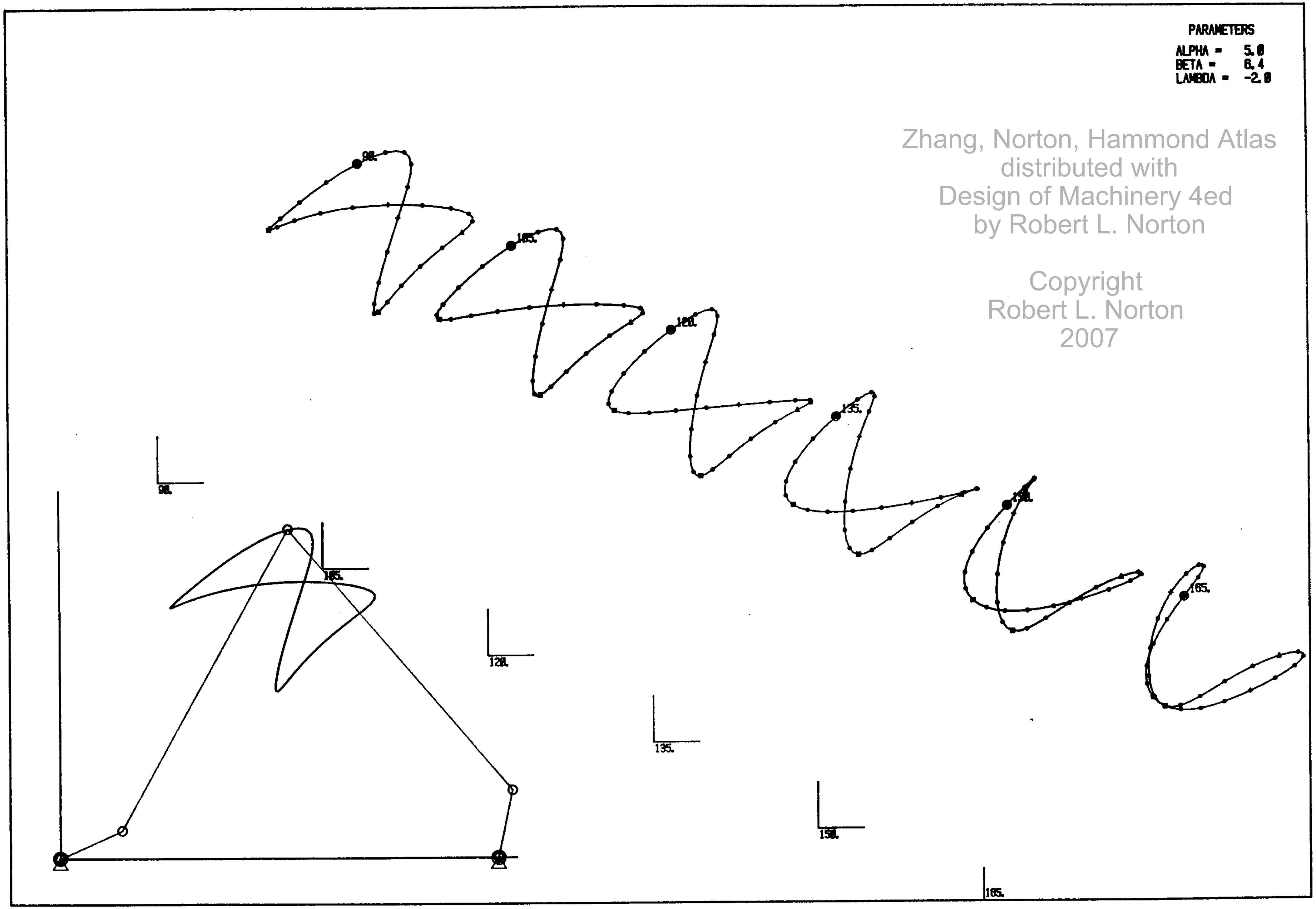
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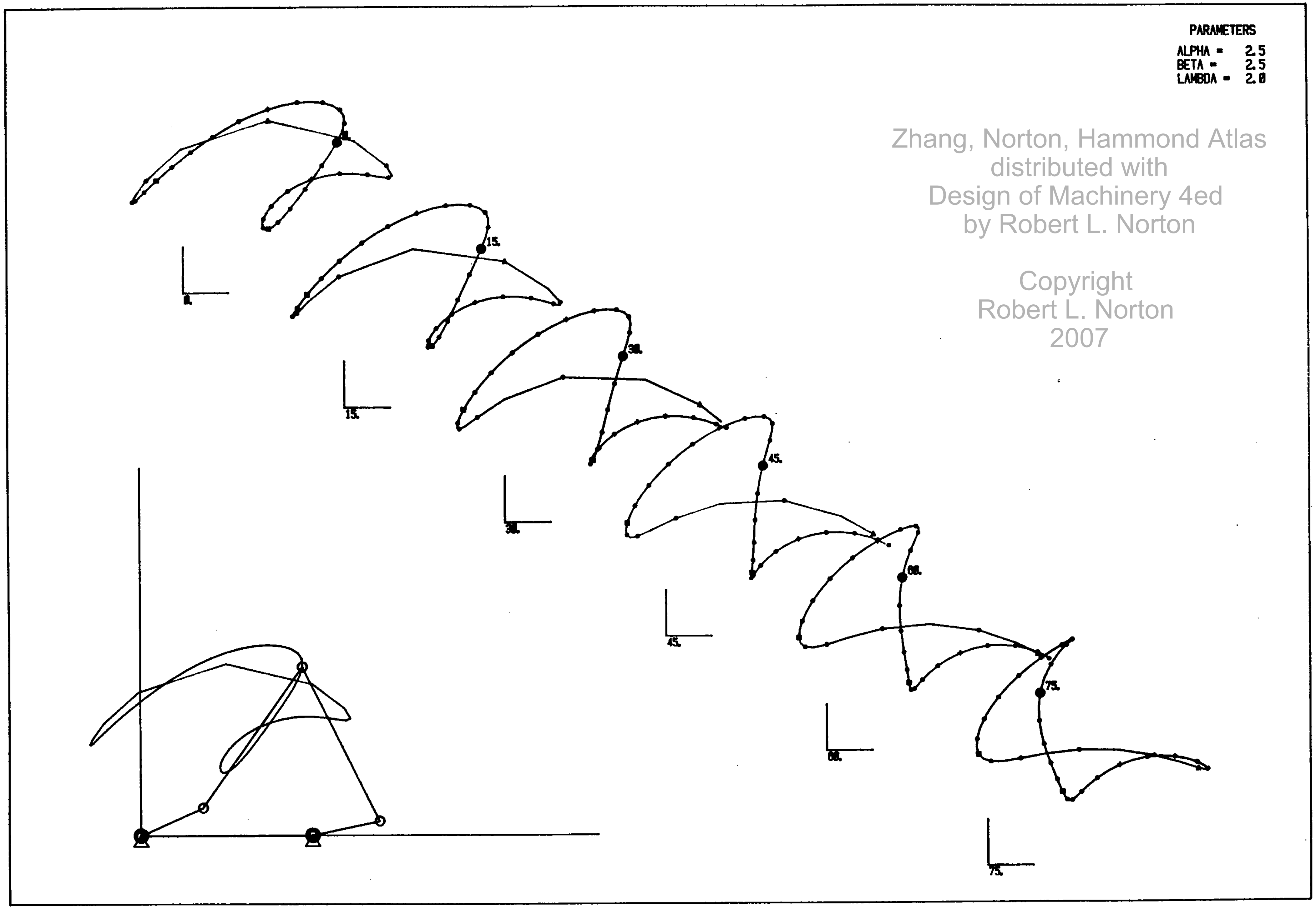
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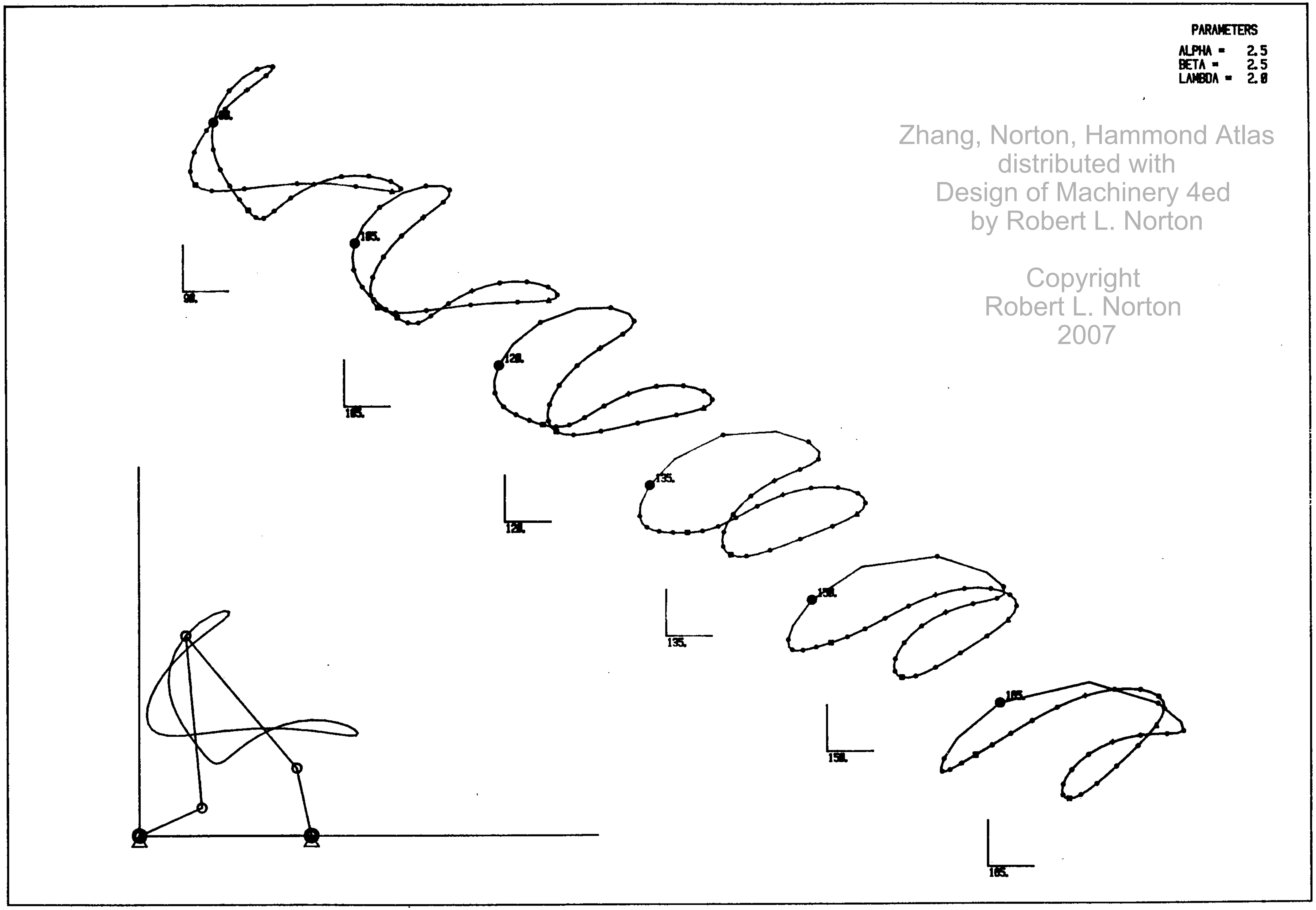

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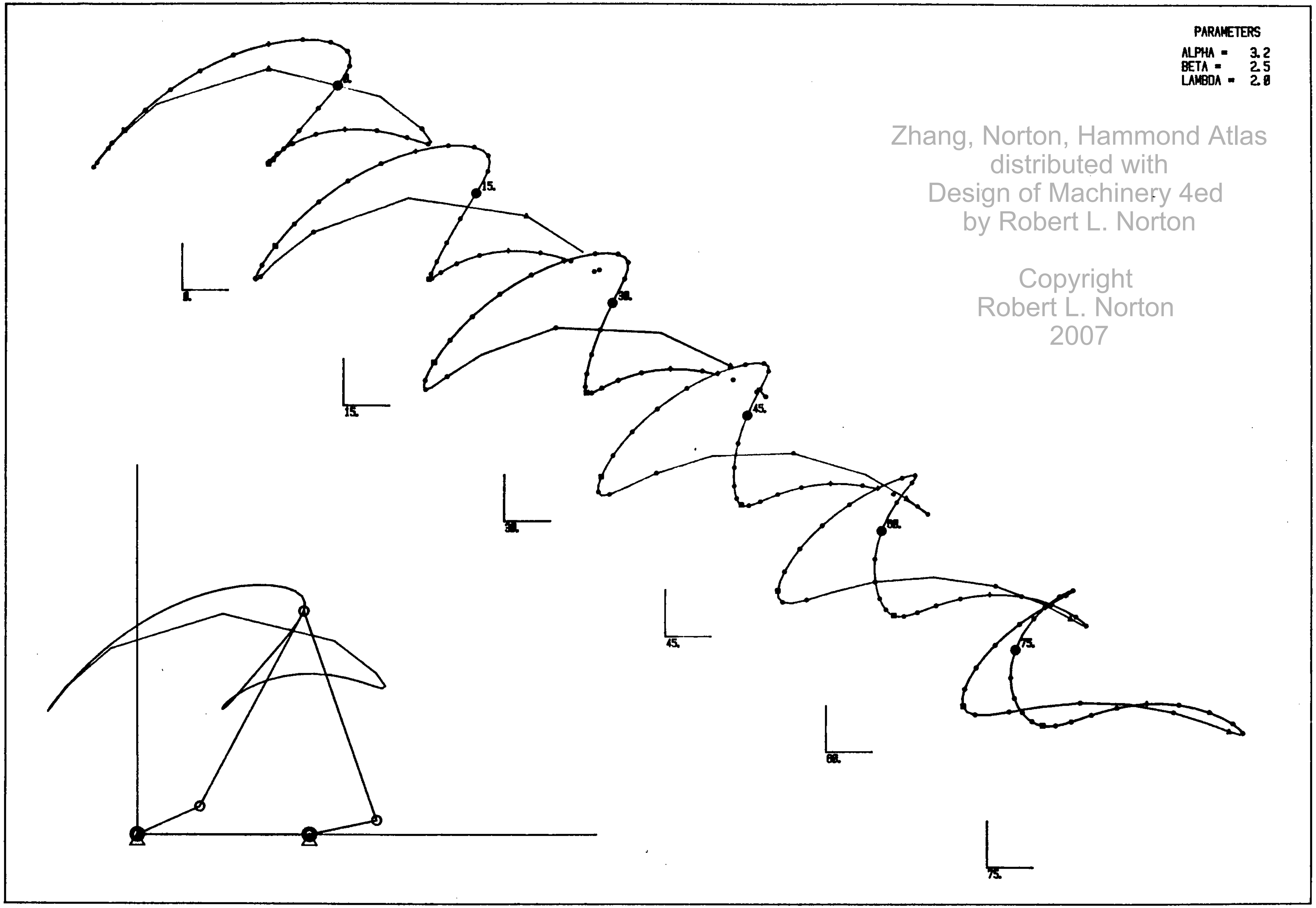
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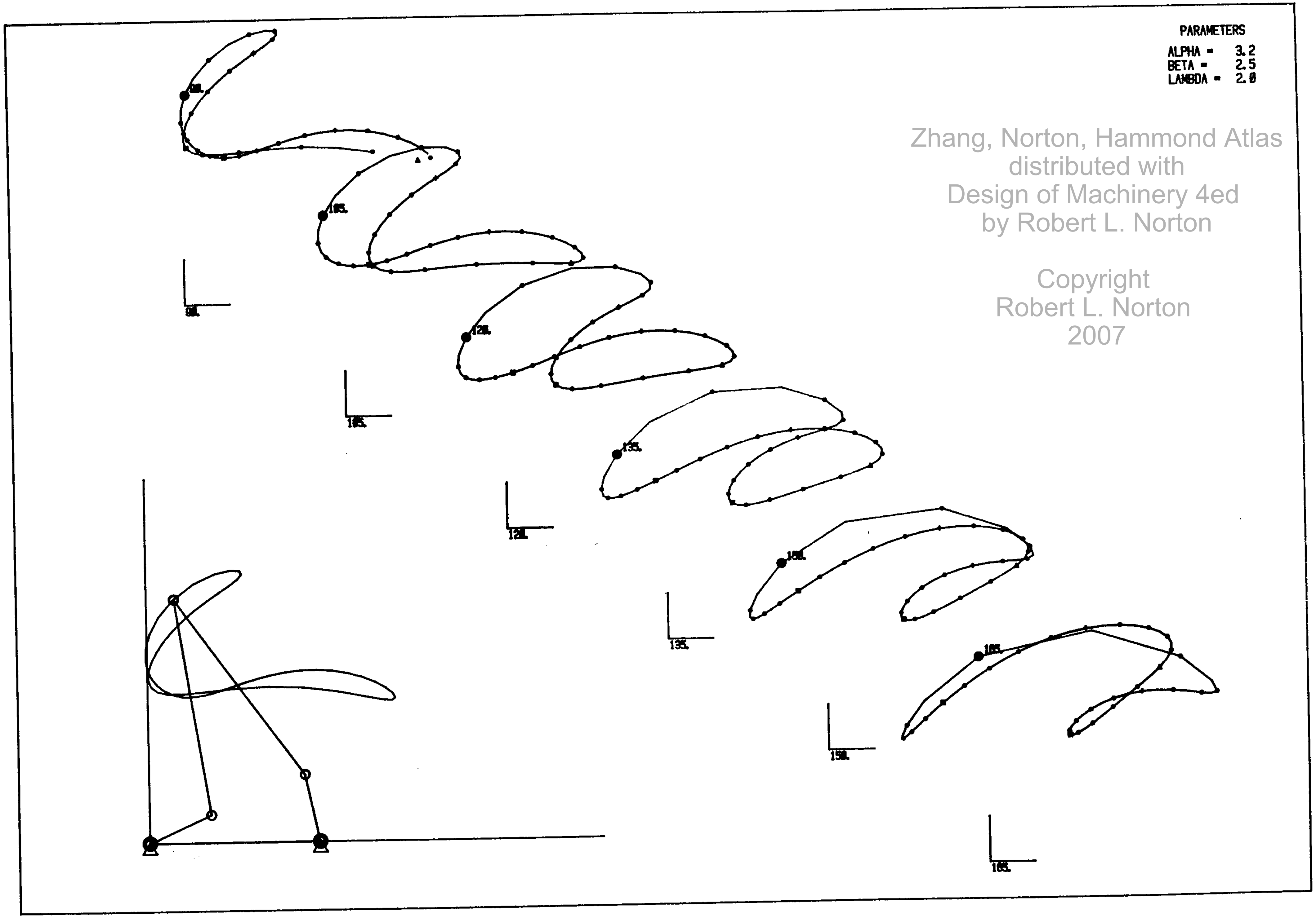
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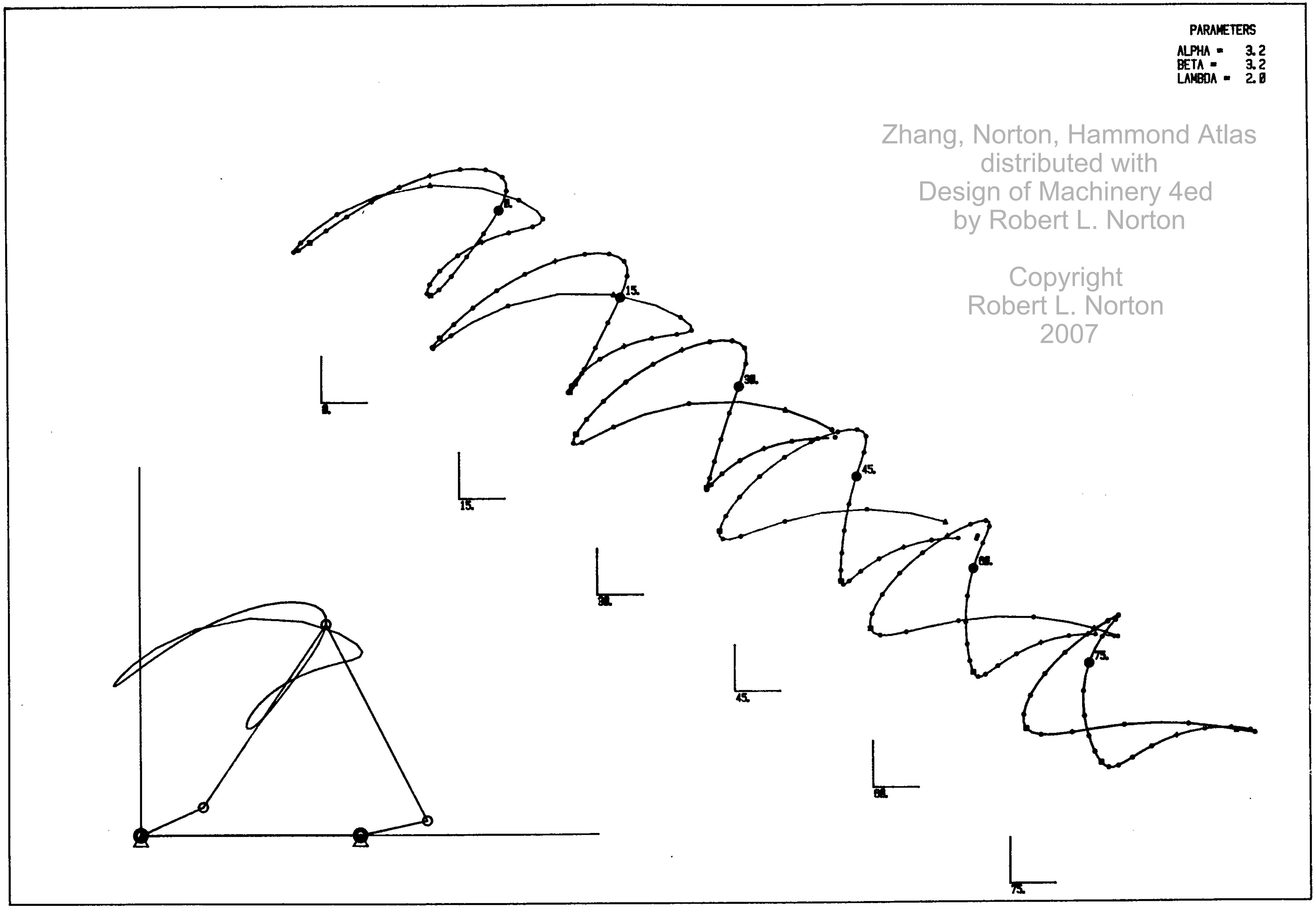
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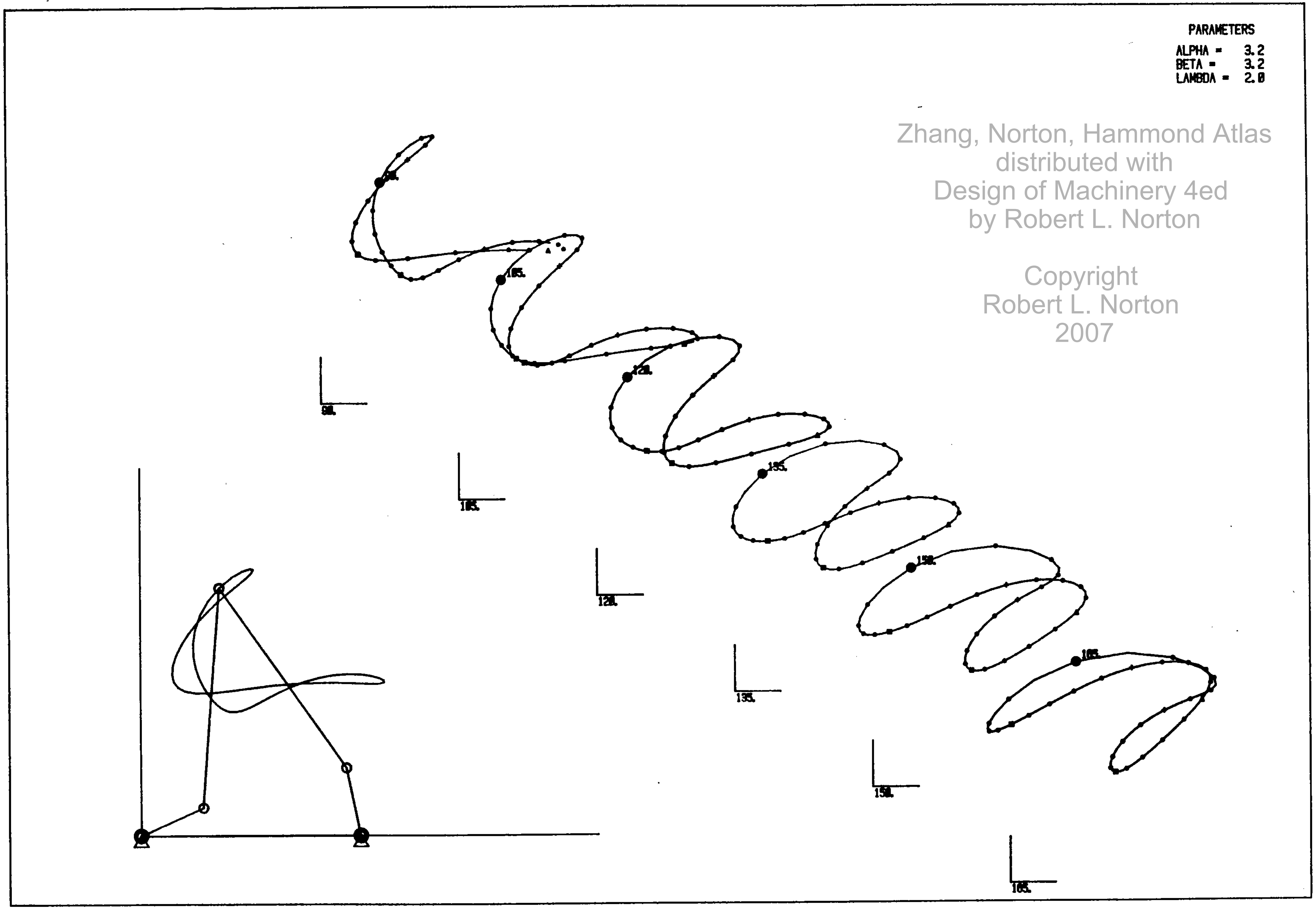

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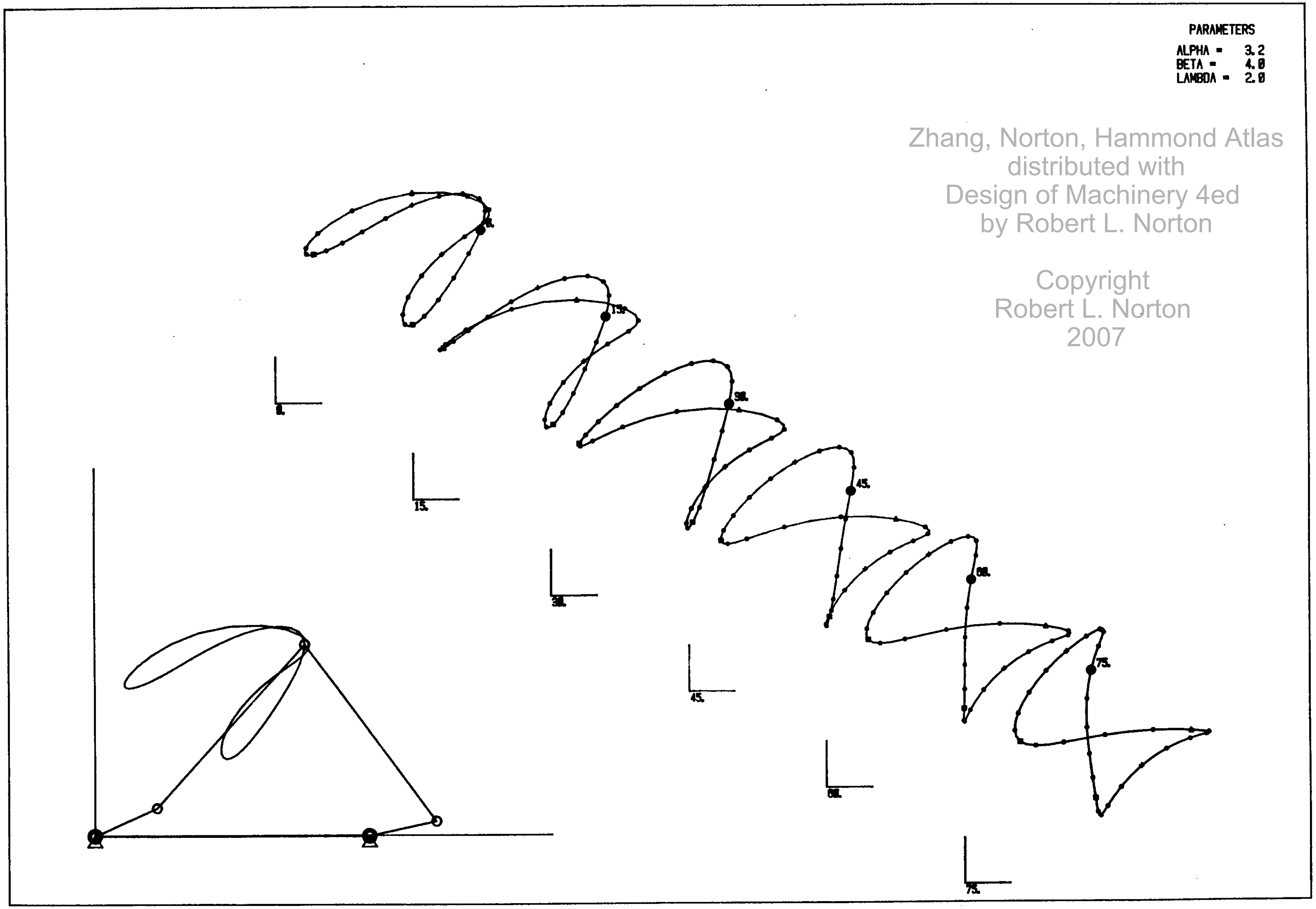
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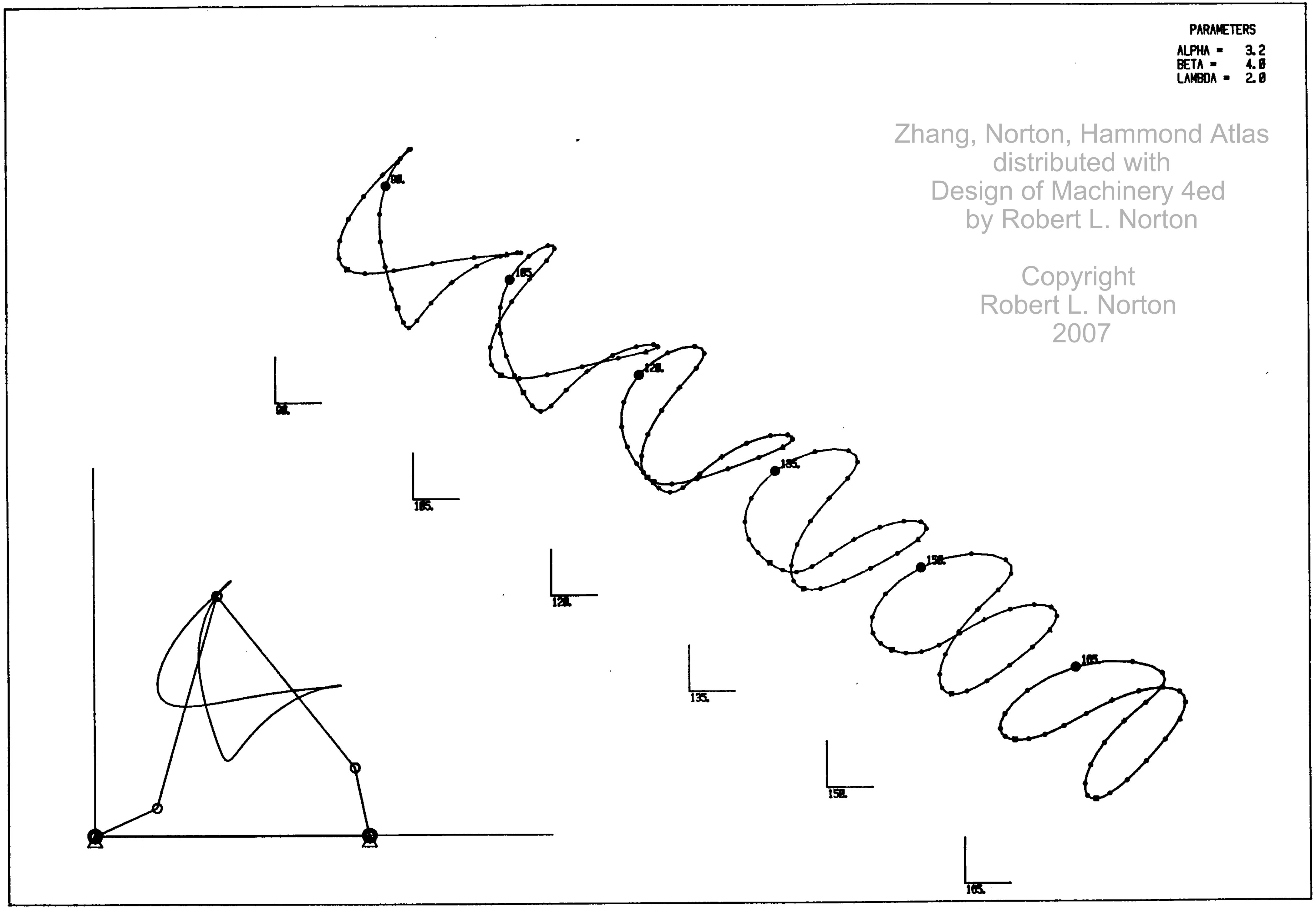
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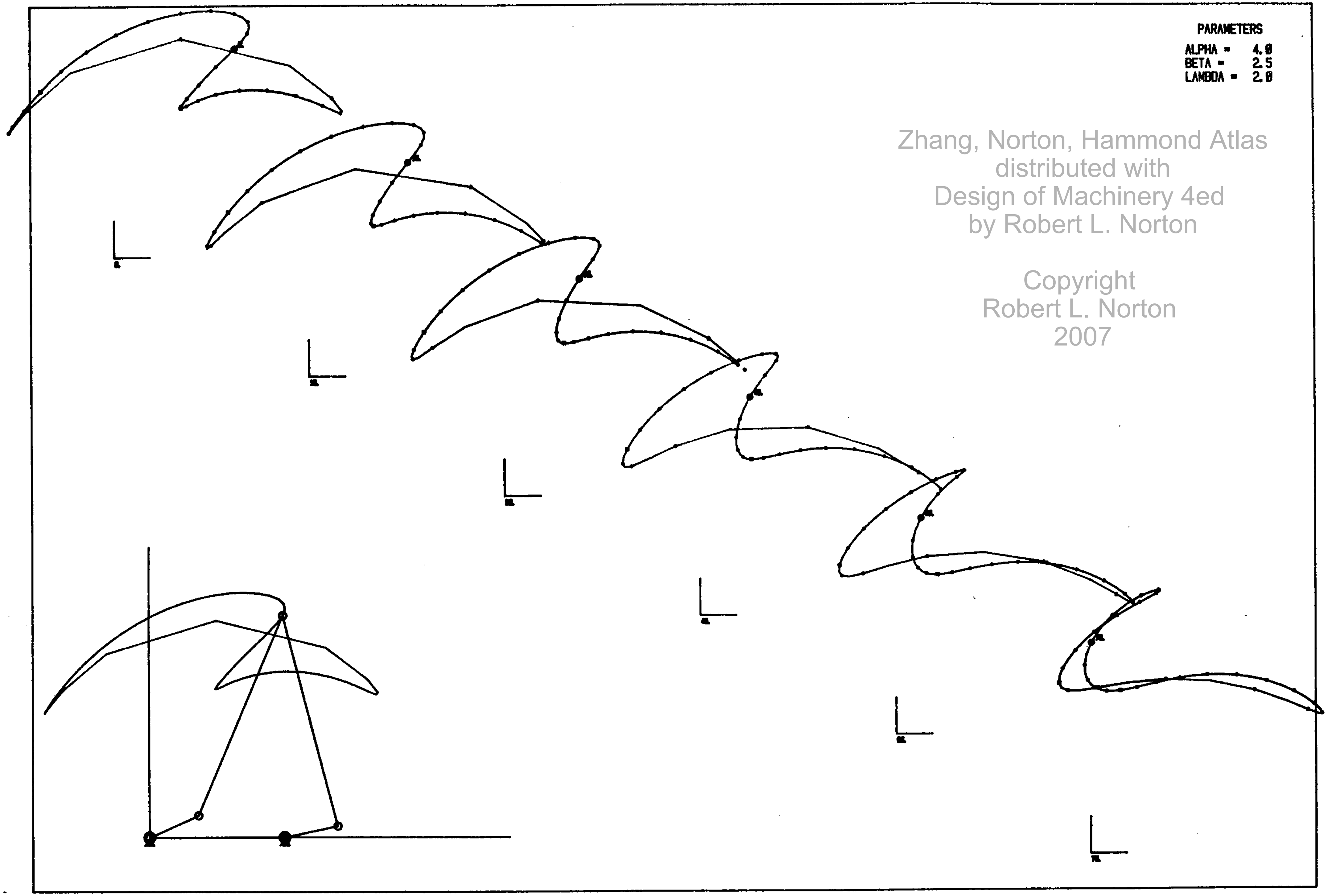

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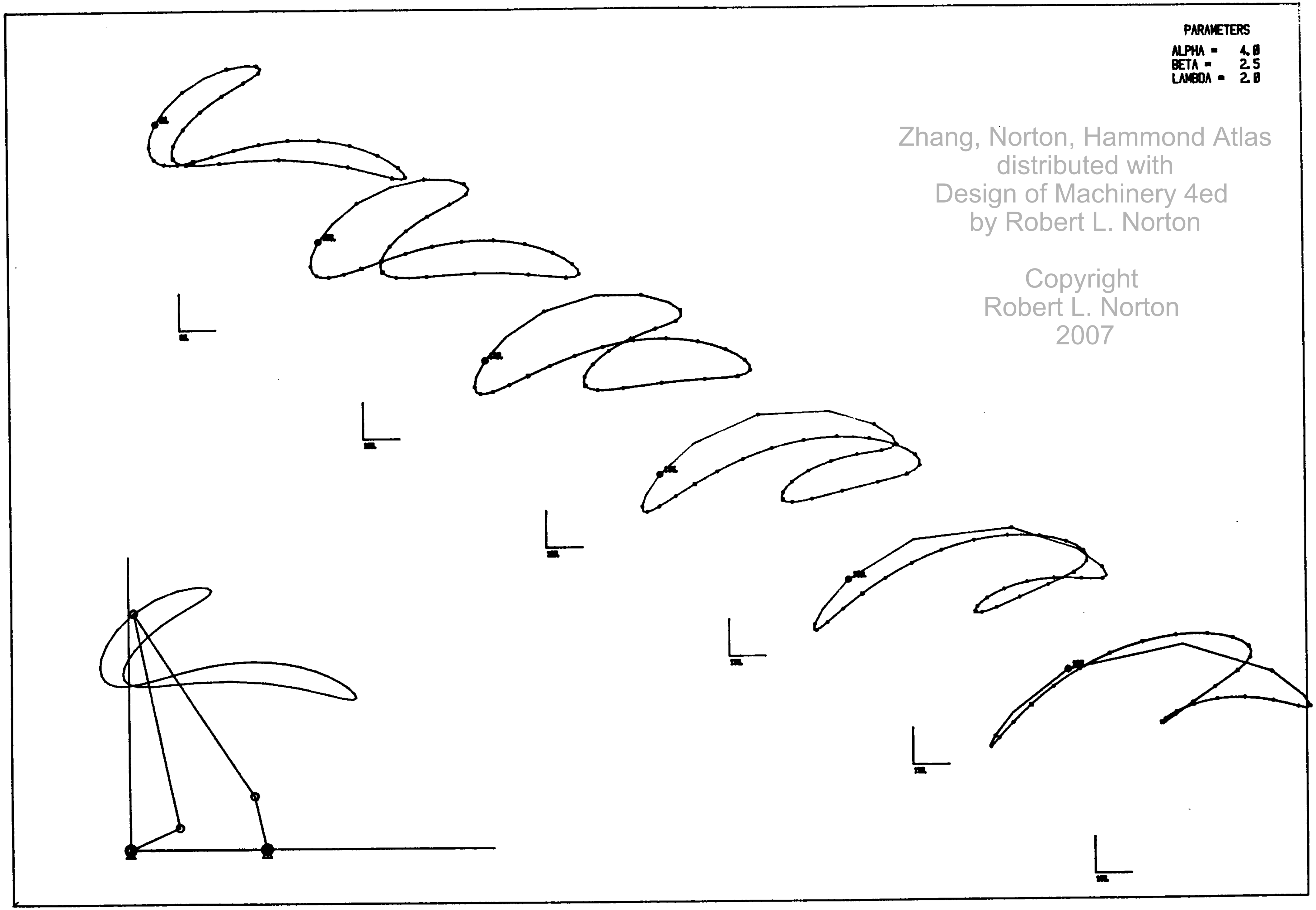
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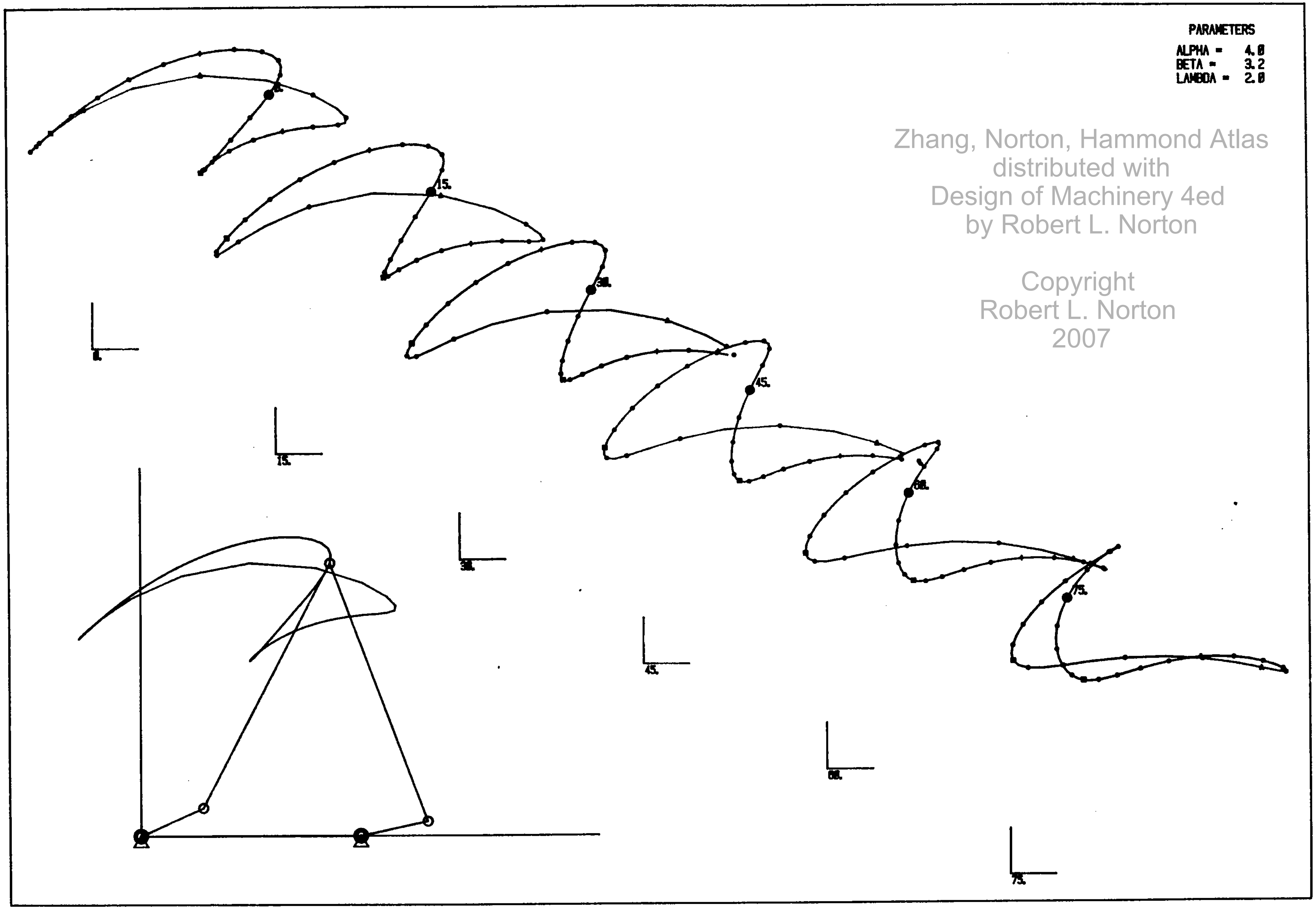
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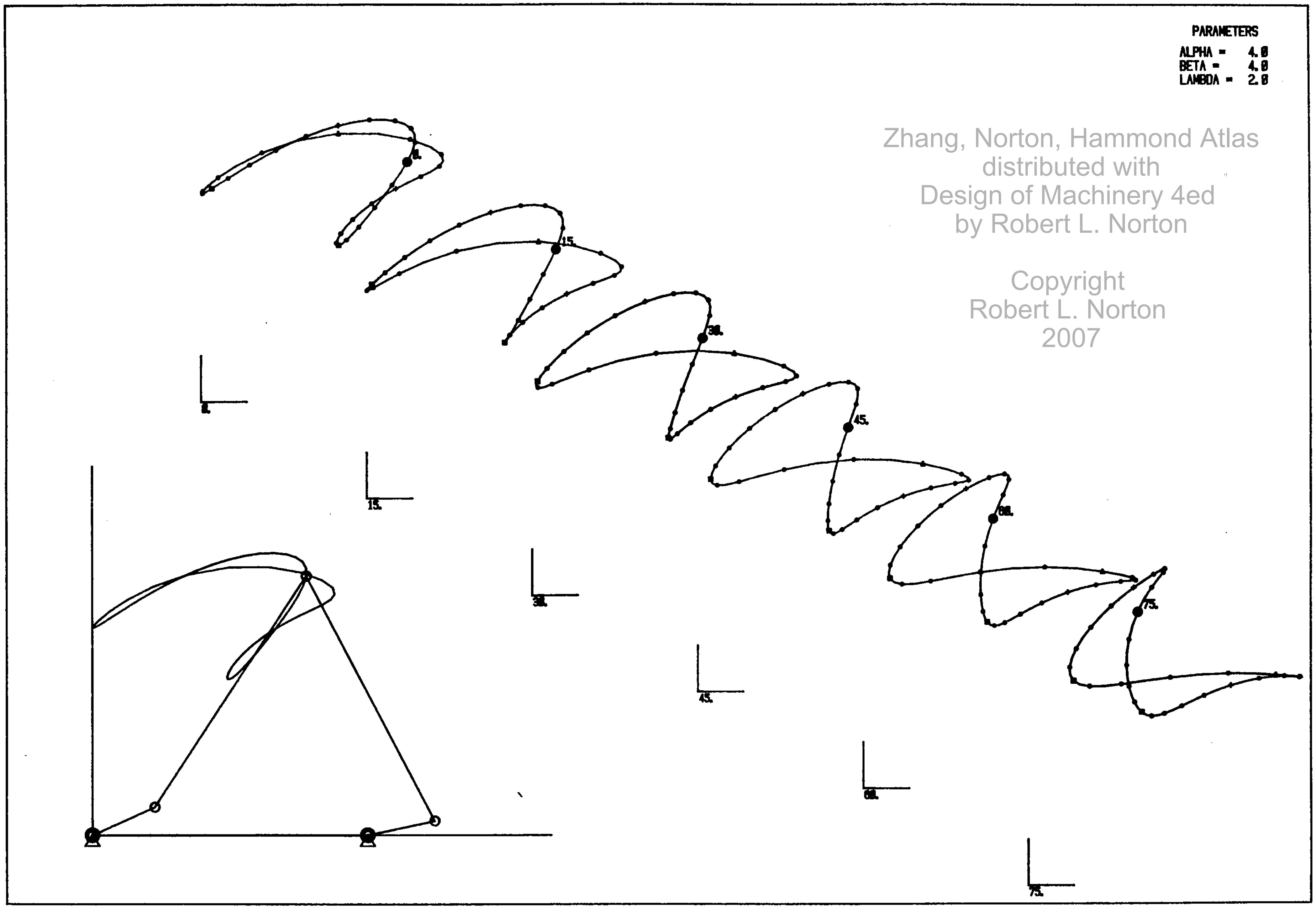
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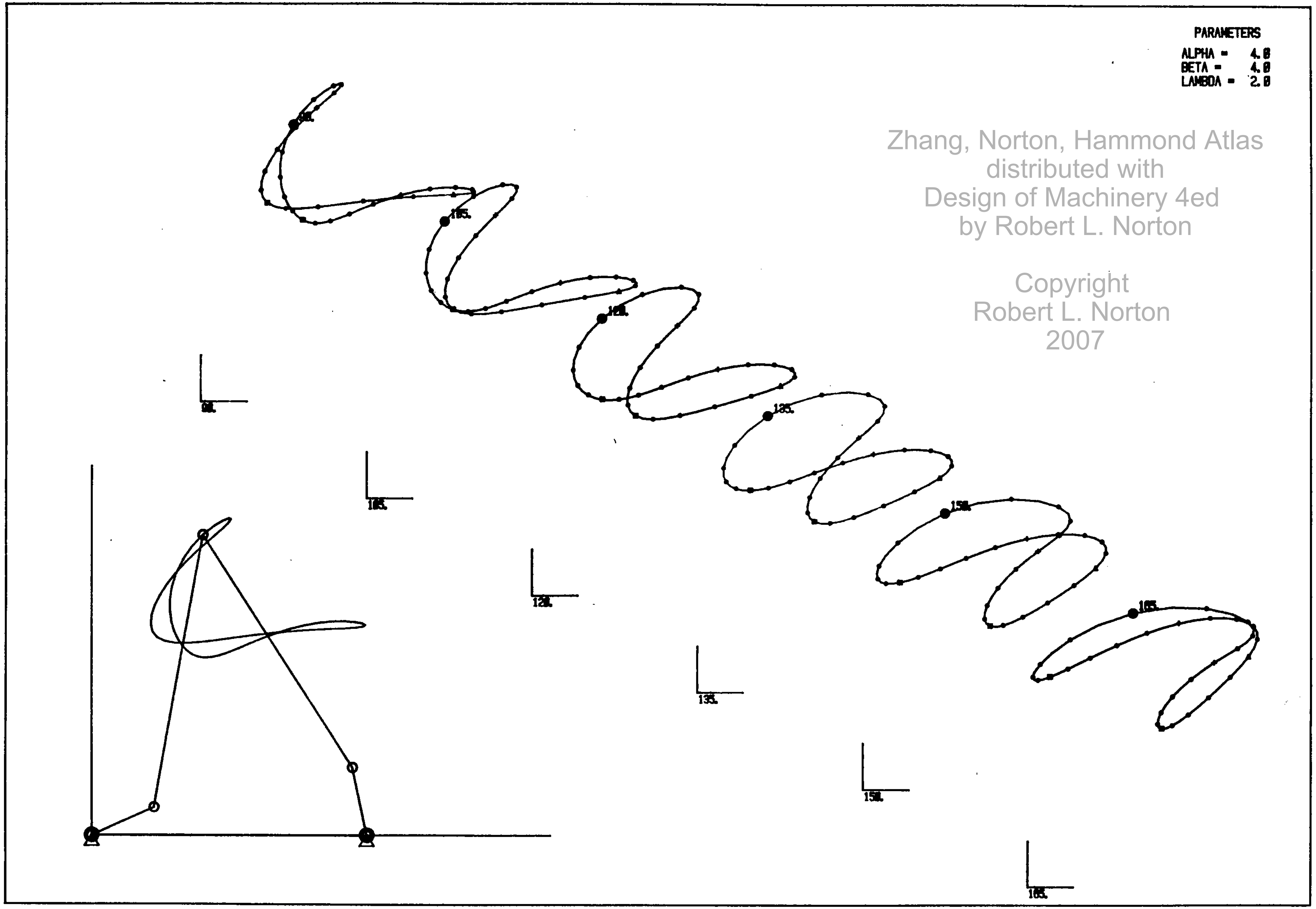
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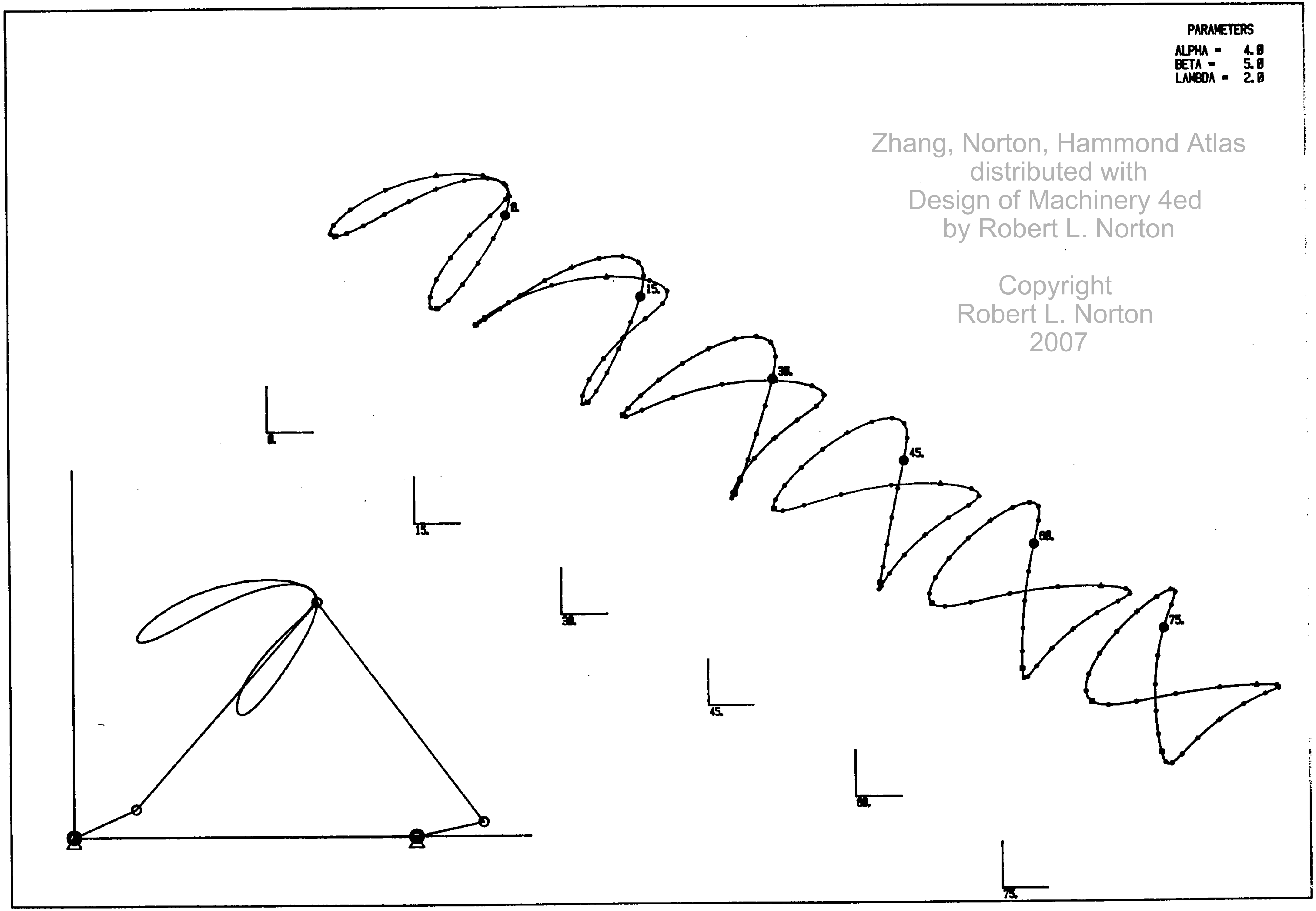
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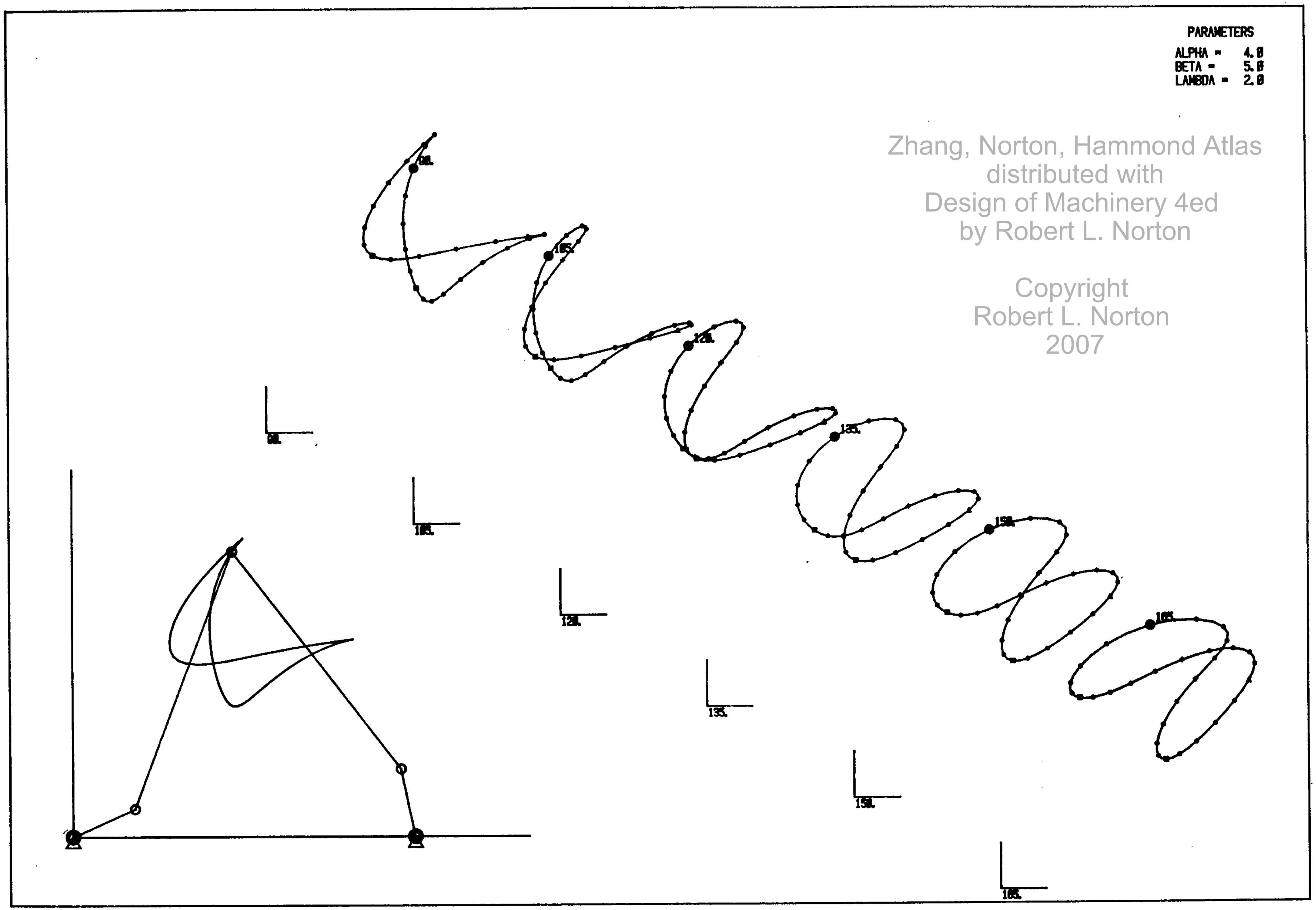
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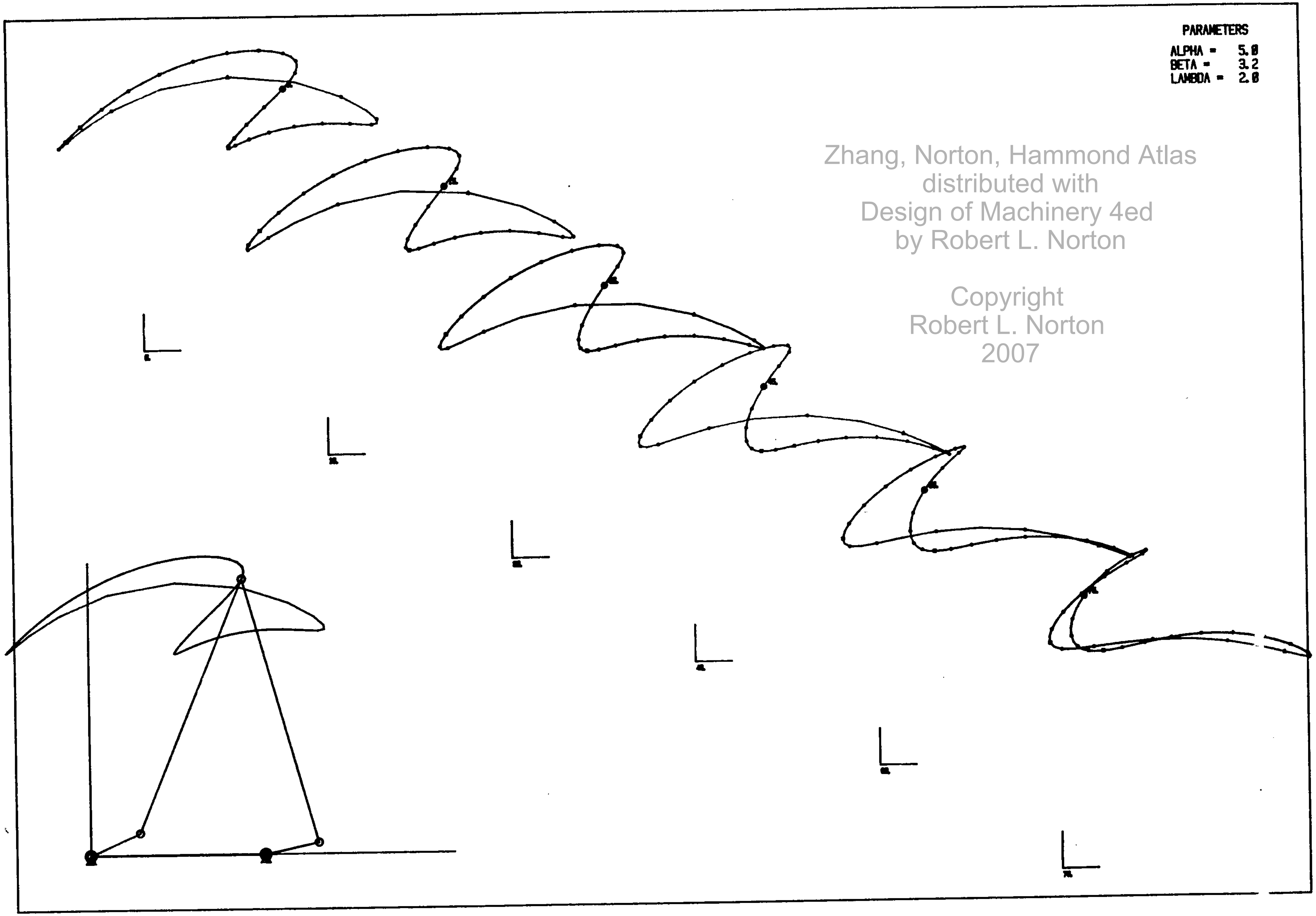
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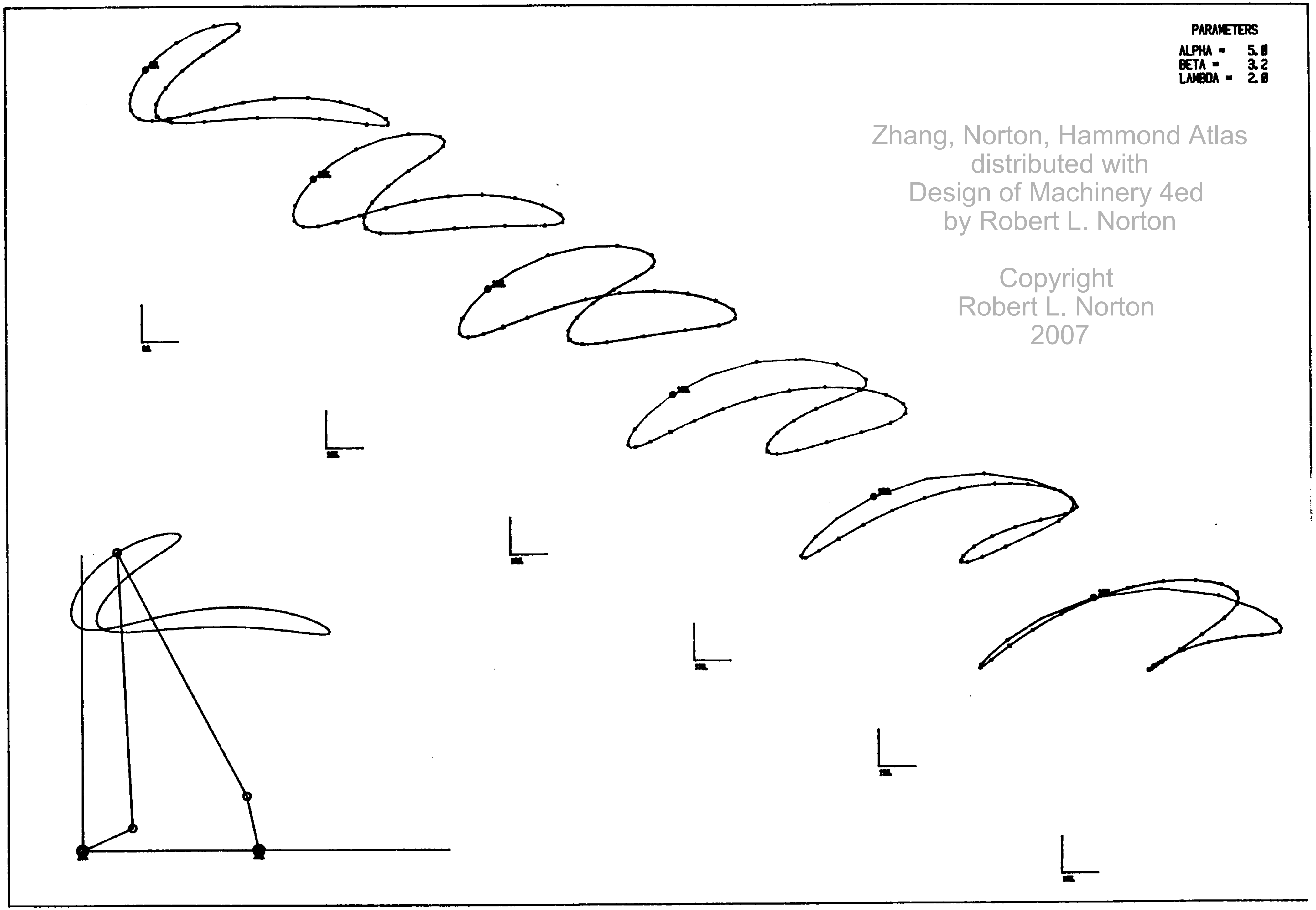
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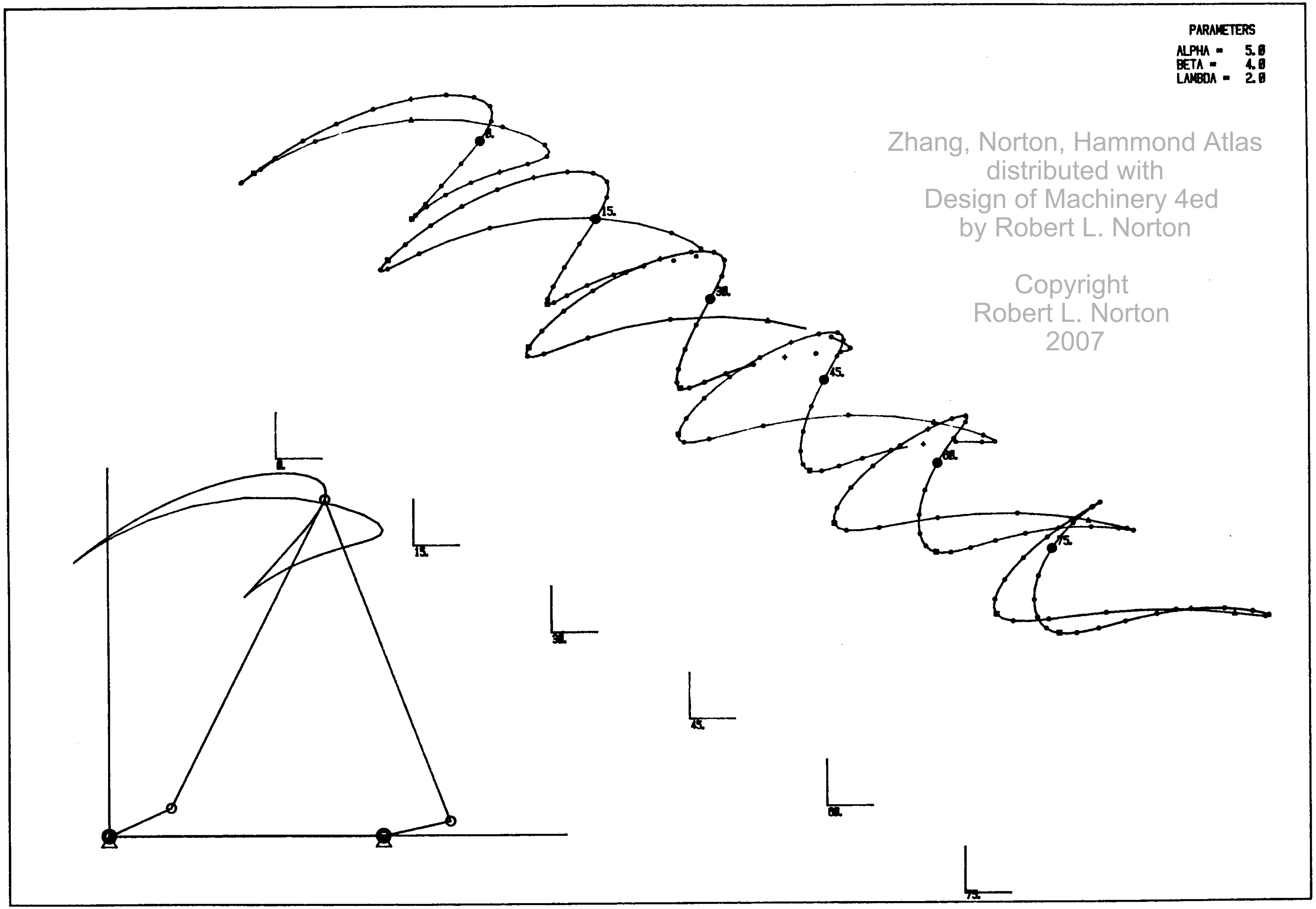
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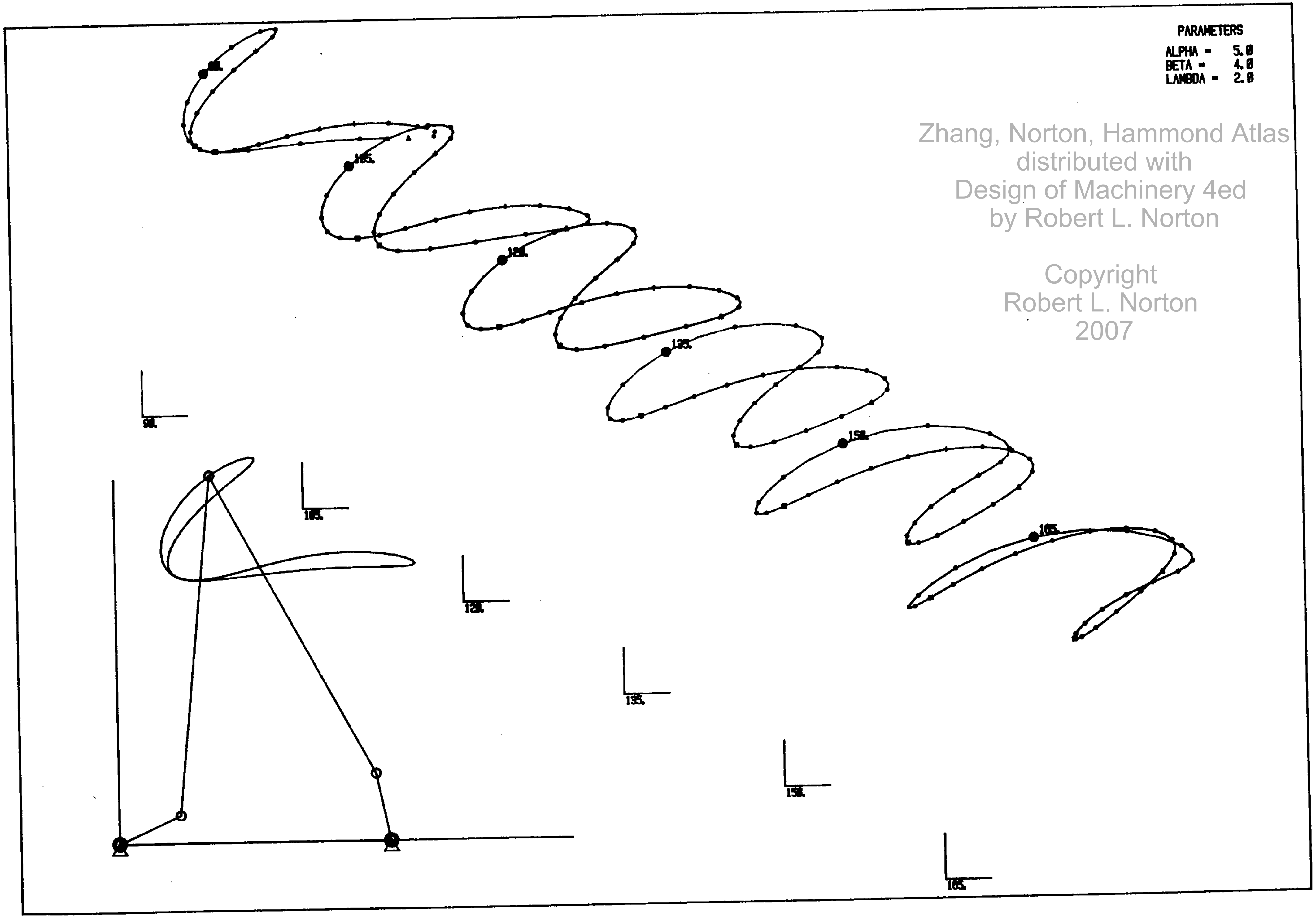
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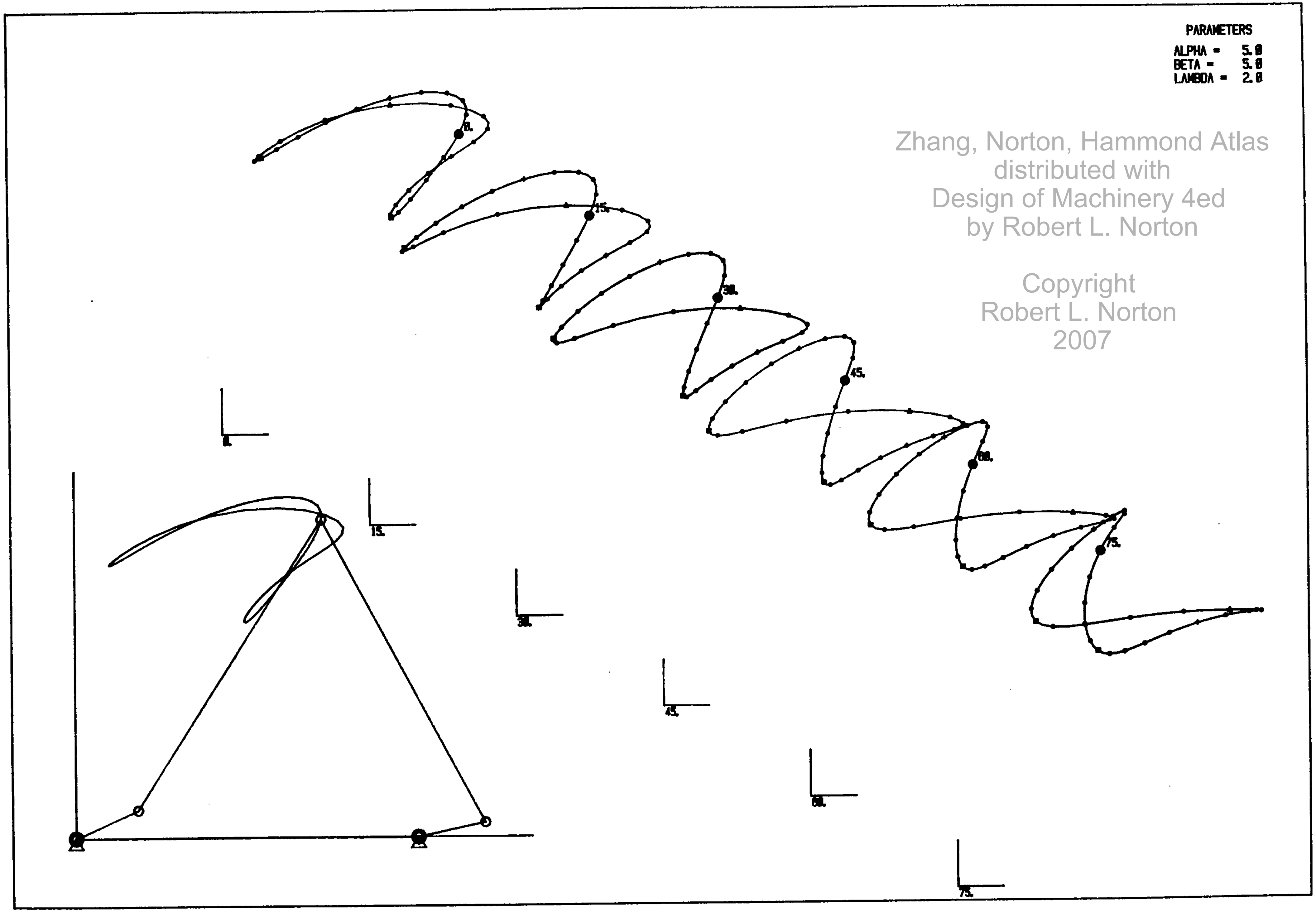
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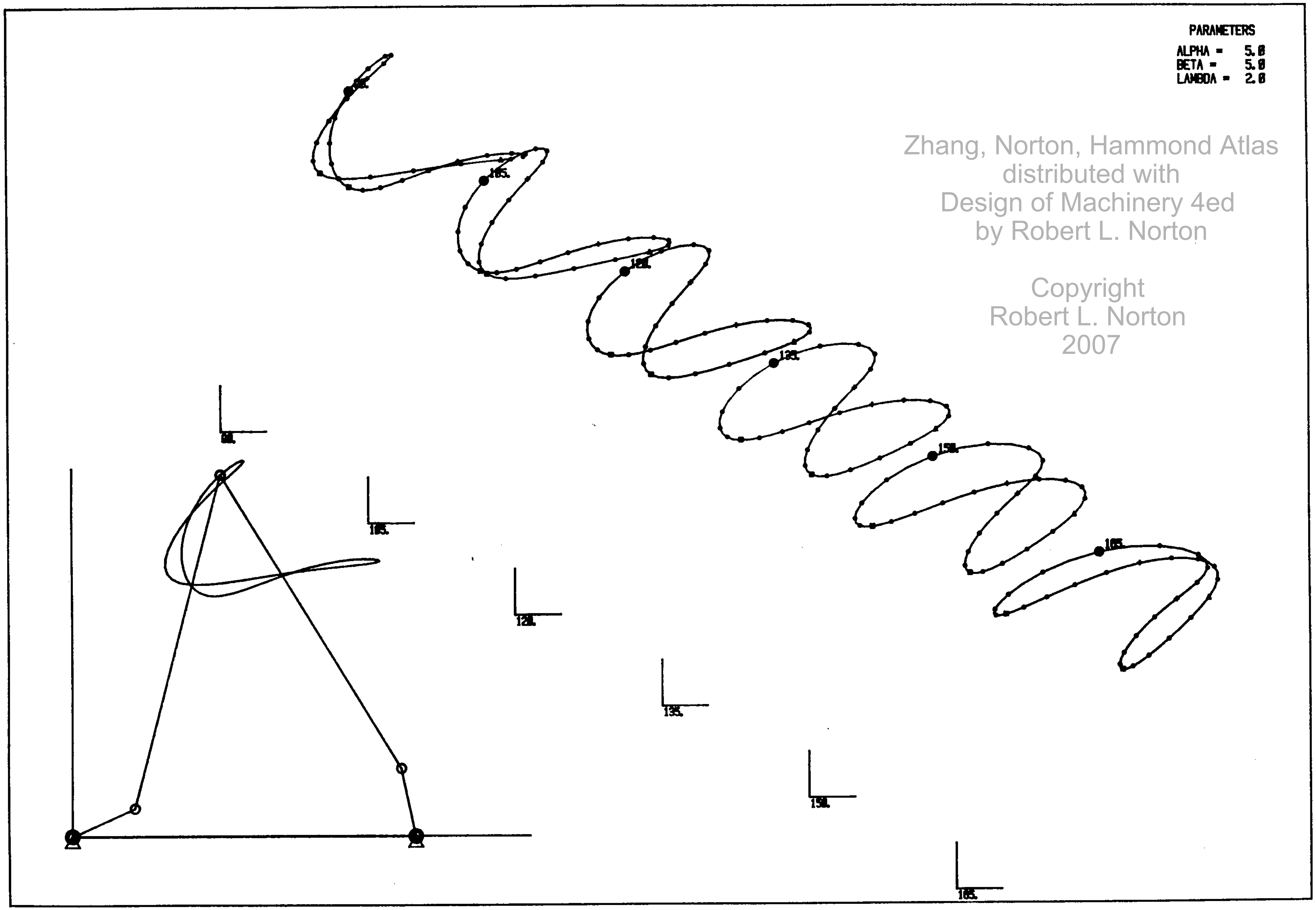
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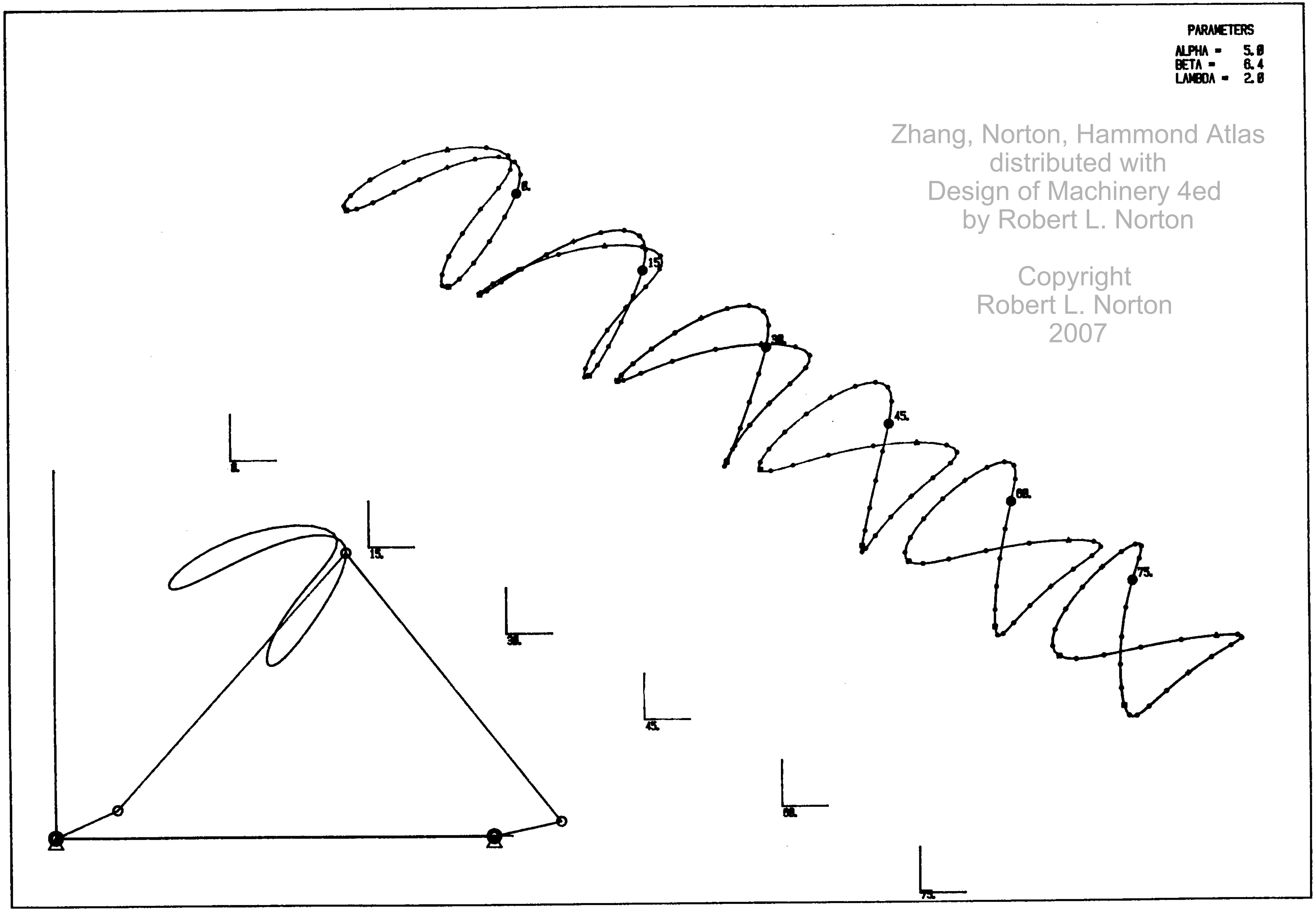
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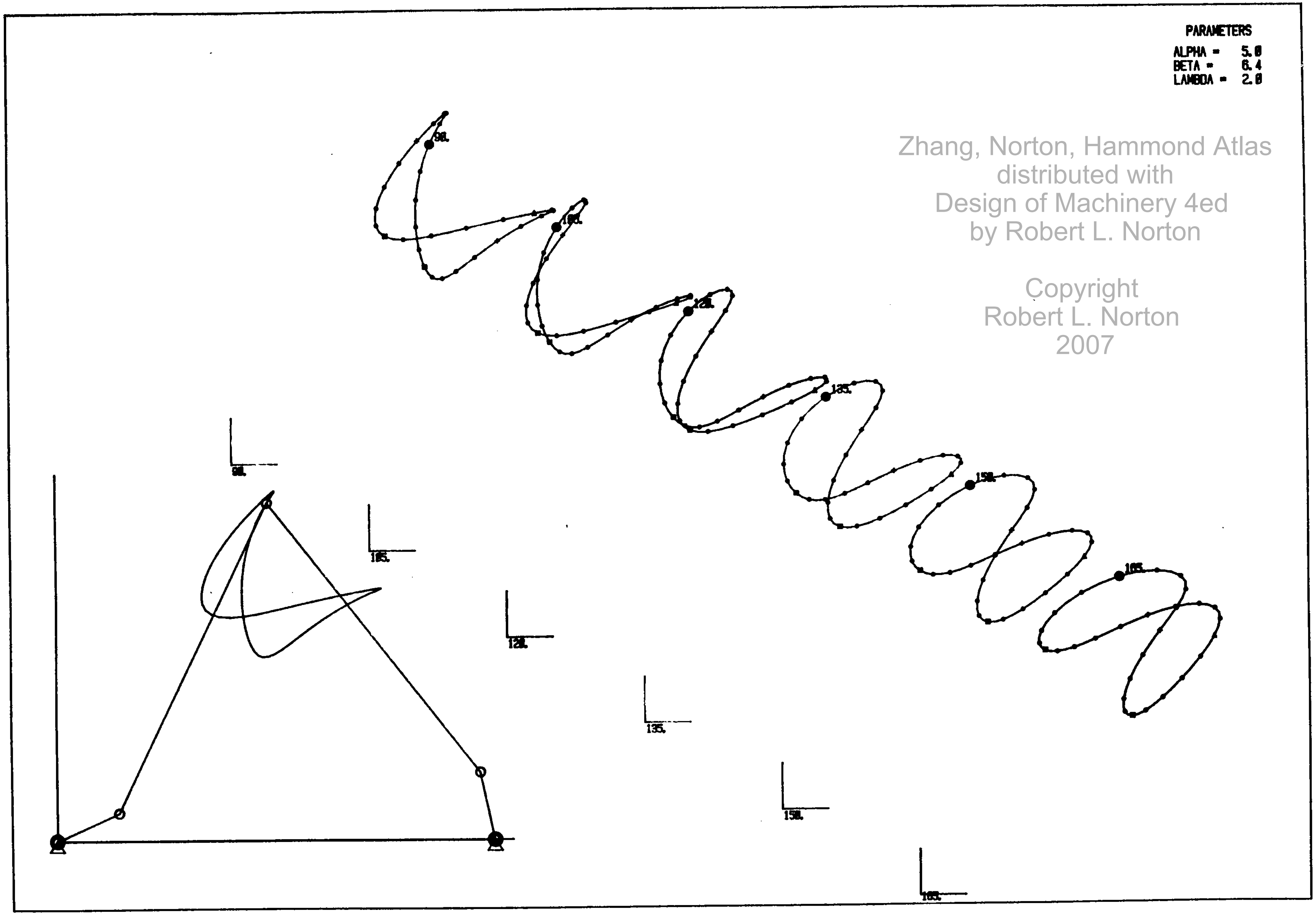
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