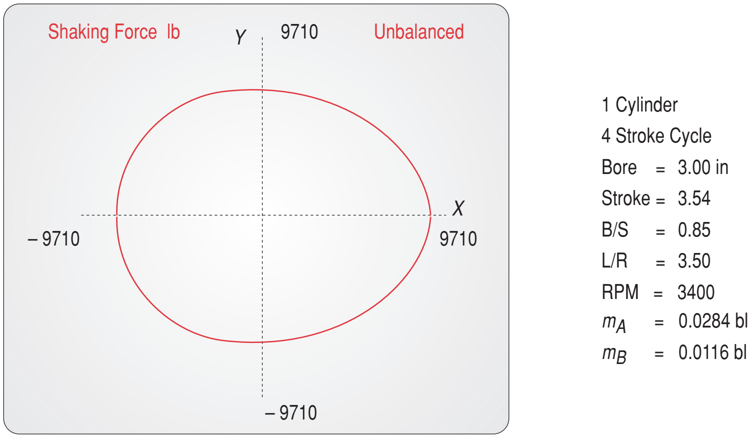

FIGURE 13-13

Shaking force in an unbalanced slider-crank linkage

13.6 INERTIA AND SHAKING TORQUES

The **inertia torque** results from the action of the inertia forces at a moment arm. The inertia force at point A in Figure 13-12 (p. 680) has two components, radial and tangential. The radial component has no moment arm. The tangential component has a moment arm of crank radius r. If the crank ω is constant, the mass at A will not contribute to inertia torque. The inertia force at B has a nonzero component perpendicular to the cylinder wall except when the piston is at TDC or BDC. As we did for the gas torque, we can express the inertia torque in terms of the couple $-\mathbf{F}_{i_{41}}$, $\mathbf{F}_{i_{41}}$ whose forces act always perpendicular to the motion of the slider (neglecting friction), and the distance x, which is their instantaneous moment arm (see Figure 13-12, p. 680). The inertia torque is:

$$\mathbf{T}_{i_{21}} = \left(F_{i_{41}} \cdot x\right)\ \hat{\mathbf{k}} = \left(-F_{i_{14}} \cdot x\right)\ \hat{\mathbf{k}} \tag{13.15a}$$

Substituting for $F_{i_{14}}$ (see Figure 13-12b, p. 680) and for x (see equation 13.3a,. p. 671) we get:

$$\mathbf{T}_{i_{21}} \cong -\left(-m_B\,\ddot{x}\tan\phi\right)\left[l - \frac{r^2}{4l} + r\left(\cos\omega t + \frac{r}{4l}\cos 2\omega t\right)\right]\hat{\mathbf{k}} \tag{13.15b}$$

We previously developed expressions for $\ddot{x}$ (equation 13.3e, p. 671) and $\tan\phi$ (equation 13.7d, p. 675) which can now be substituted.

$$\begin{aligned}\mathbf{T}_{i_{21}} \cong\ & m_B\left[-r\omega^2\left(\cos\omega t + \frac{r}{l}\cos 2\omega t\right)\right]\\ & \cdot\left[\frac{r}{l}\sin\omega t\left(1 + \frac{r^2}{2l^2}\sin^2\omega t\right)\right]\\ & \cdot\left[l - \frac{r^2}{4l} + r\left(\cos\omega t + \frac{r}{4l}\cos 2\omega t\right)\right]\hat{\mathbf{k}}\end{aligned} \tag{13.15c}$$